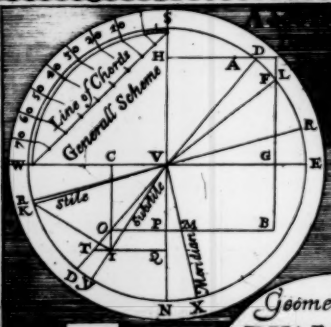

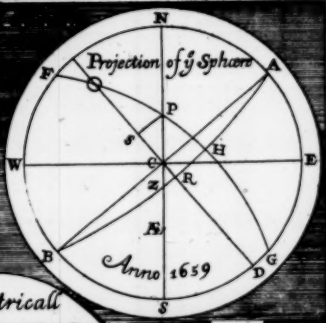

Geometricall

DYALLING

or

Dyalling by a line of

CHORDS

onely

by JOHN COLLINS

Accomptant Philomath

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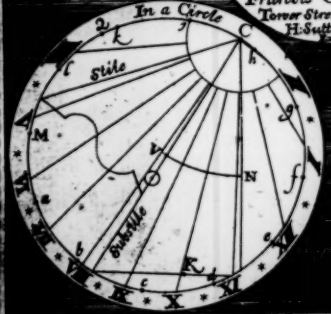

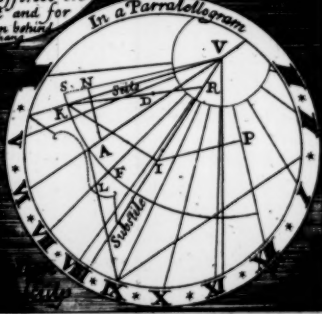

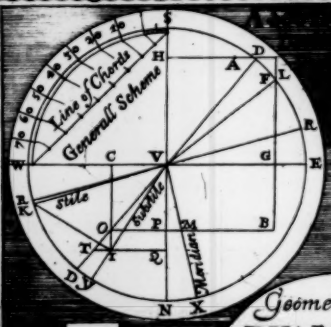

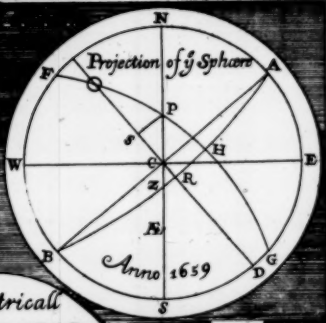

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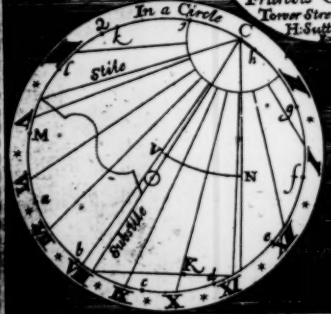

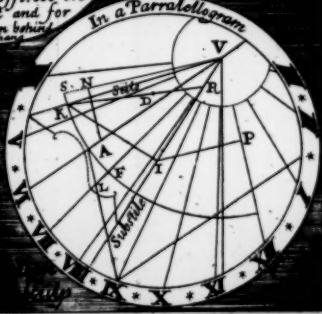

Geometricall Dyalling:

OR,

DYALLING

Performed by a Line of CHORDS onely,

Or by the

PLAIN SCALE.

Wherein is contained two ſeveral Methods of Inſcribing the Hour-Lines in all Plains, with the Subſtile, Stile and Meridian, in their proper Coaſts and Quantities;

Being a full Explication and Demonſtration of divers difficulties in the Works of Learned Mr. *Samuel Foſter* deceaſed, late Profeſſor of Aſtronomy in *Greſham Colledge*, ; Alſo a Collection of divers things from the Works of *Clavius* and others.

Whereto is added four new Methods of Calculation, for finding the Requiſites in all Leaning Plains, with full directions ſuited to each Method for placing them in their proper Coaſts, without the help of any Delincations.

Alſo how by Projecting the Sphere, to meaſure off all the Arks found by Calculation, and to determine what hours are proper to all kinde of Plains, omitting ſuperfluity.

Laſtly, the making of Dyals from three Shadows of a Gnomon placed in a Wall at random, with a Method of Calculation ſuited thereto; and divers ways from three Shadows, to finde a Meridian-line.

Written by *John Collins* of *London* Accomptant, *Philomath.*

London, Printed by *Thomas Johnſon* for *Francis Coſſinet*, and are to be ſold at his Shop at the Anchor and Mariner in Tower-ſtreet, at the end of Mincing-lane, with other Mathematical Books; Alſo to be ſold by *Henry Sutton* Mathematical Inſtrument-Maker, living in Threed-needle Street behinde the Exchange. 1659.

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To the Reader.

BEing in conference with my loving friend Mr. *Thomas Rice*, one of the Gunners of the Tower, much exercised in making of Dyals in many eminent places of the City, he was pleased to communicate unto me the knowledge of a General Scheme, for Inscribing the Requisites in all Oblique Leaning Plains, which he added, was the useful Invention of the Learned Master *Samuel Foster*, Professor of Astronomy in *Gresham* Colledge *London*, deceased, from whom he received Instructions concerning the same, in the year 1640. And the said Mr. *Rice* not having his Papers about him, did dictate to me from his Memory the Construction and Practice of the said Scheme, which I afterwards Methodized just as it is delivered in *page* 25 and 26. At the same time also I received directions for Inscribing the Substile and Stile in upright Decliners, and East or West Reclining, or Inclining Plains, but they were of another mans Invention, and did not seem to be derived from the former general Scheme, and therefore I have not used them in this Treatise, but derived the performance thereof from the said general Scheme: Moreover, Mr. *Rice* added, that in regard of the death of the Author, and since of his Executor, who had the care and inspection of his Papers, I should do well to Study out the Demonstration of the former Scheme, and make it publick, the rather, because it hath been neglected. The manner of Inscribing the hour-lines being already published in a Treatise of the Authors, Intituled, *Posthumi Posteri*; this desire of his, which was also furthered by Mr. *Sutton* and others, I am confident is fully effected in the following Treatise, and much more then was in his Request, and here

here let me adde, that I have had no other light from the Endeavors of the Learned Author, then what was as above communicated unto me, all which might be expreſſed in half a page of paper, or little more, to which I have made this large Acceſs and Collection, *nam facile eſt inventis adore*; not that I would hereby any thing endeavor to eclipſe the Authors Work, the excellency of whoſe Inventions in this and other kindes, will ſpeak forth his Renown to all poſterity. And though in probability I have not performed ſo much, nor ſo well, as was obvious to the knowledge of the Learned Author; yet I am confident when the Reader underſtands what is written, he would be as loath to be without the knowledge thereof, as my ſelf; and I am induced to believe, that the Author left nothing written about many particulars in this Treatiſe; throughout which, we ſuppoſe the Reader, furniſhed with the common Rudiments of Geometry, that he can raiſe Perpendiculars, draw Parallels, deſcribe a Parralellogram, bring three points into a Circle, underſtands Definitions and Tearms of Art, knows what a Line of Chords is, can prick off an Arch thereby; all which, with Delineations for all the uſual Caſes of Triangles, both from Projection and Proportions, the Reader will meet with in my Treatiſe, called, *The Mariners Plain Scale new plain'd*, now in the Preſs. *Vale & fruere.*

I remain thy friend, and a Well-willer to the Publique Advancement of Knowledge,

JOHN COLLINS.

The

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Characters used in this Book.

\+ *Plus, for more, or Addition.* — *Minus less, or Substraction.* = *Equal.* q *For square.* □ *Square.* ▭ *The Rectangle or Product of two terms.* :: *for Proportion.*

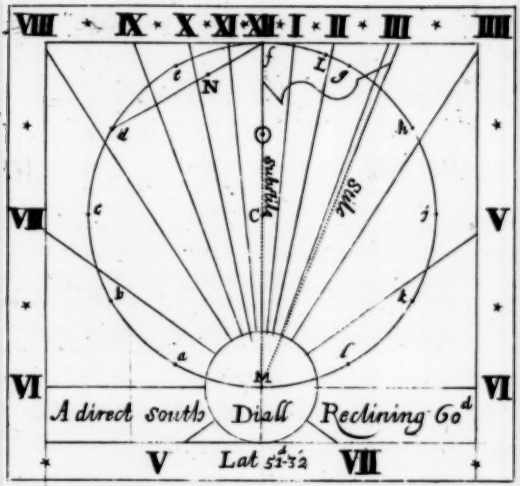
If the Brass Prints in this Book be thought troublesome to binde up, they may be placed at the end thereof, for the Pages to which they relate, are graved upon them.

A Quadrant of a Circle being divided into 90d, all that is required in this Treatise, is to prick off any number of the said degrees with their Sub-divisions, which may be easily done any where from a Quadrant drawn and divided into nine equal Parts, and one of those parts into ten sub-divisions, called degrees, but for readiness the equal divisions of a whole Quadrant are transferred into a right line (as in the Frontispiece) called a Line of Chords, serving more expedisly to prick off any number of degrees or minutes in the Arch of a Circle.

An Advertisement.

The Reader may possibly desire to be furnished with such Scales to any Radius, these and all manner of other Mathematical Instruments either for Sea or Land, are exactly made in Brass or Wood, by *Henry Sutton* in Threed-needle-street, behinde the Exchange, or by *William Sutton* in Upper Shadwell, a little beyond the Church, Mathematical Instrument-Makers.

THE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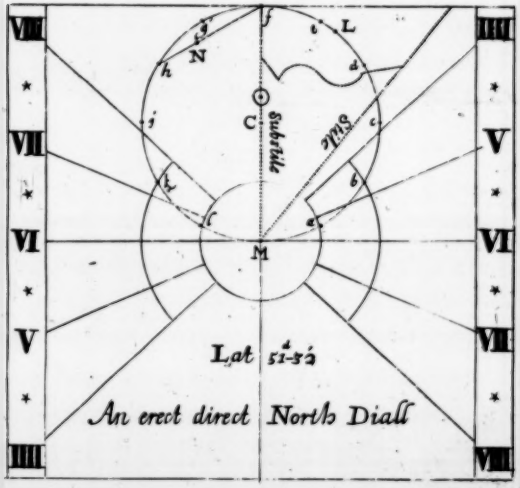
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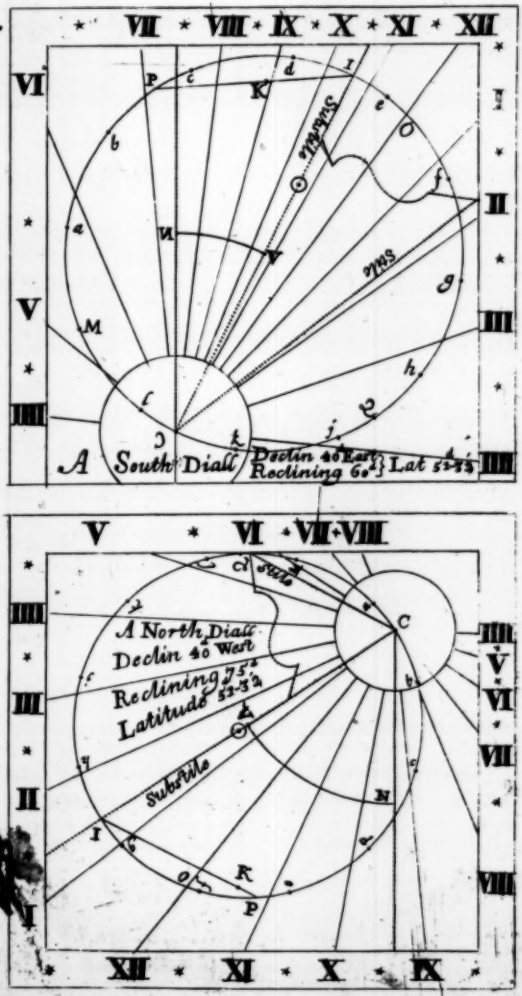
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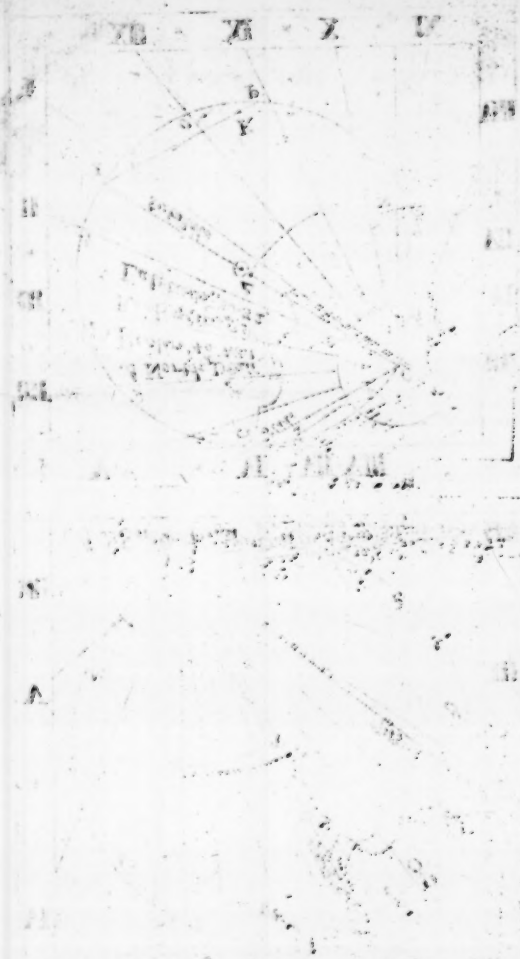


THE DISTINCTION OF DYALS.

THough much of the ſubject of Dyalling hath been wrote already, by divers in diverſe Languages, notwithſtanding the Reader will meet with that in this following Treatiſe, which will abundantly ſatisfie his expectation, as to the particulars in the Preface, not yet divulged in any Treatiſe of this nature.

In this Treatiſe, that we might not be too large, divers definitions are paſſed over, ſuppoſing that the Reader underſtands what a Dyal is, what hour Lines are, that that part of the Stile that ſhews the hour, ought to be in the Axis of the World, that the hour Lines being projected on any Regular Flat, will become ſtraight Lines.

Dyals are by *Clavius* in the ſecond Chapter of his *Gnomonicke* diſtinguiſhed into Seventeen Kindes.

1. *The Horizontal, being parallel to the Horizon.*
2. *South and North direct, by ſome called a Vertical Dyal, becauſe parallel to the prime Vertical or Circle of Eaſt and Weſt.*
3. } *South direct, Reclining or* } *North* } *leſs* } *then the Pole.*
4. } *Inclining* } *more* }
5. } *South direct, Inclining or* } *leſs* } *then the Equinoctial*
6. } *North Reclining* } *more* }

7. *South direct Reclining, or North Inclining to the Pole, called a direct Polar Plain, because Parallel thereto.*

8. *South direct Inclining, or North Reclining to the Equator, called an Equinoctial Plain, because parallel thereto.*

9. *South and North, Declining East or West.*

10. *East and West direct.*

11. *East and West direct, Reclining or Inclining.*

12. *A* { *South* / *North* } *Plain declining* { *East or West* } { *Reclining* / *Inclining* } *to the Pole, called a Polar Decliner.*

13. *A* { *South* / *North* } *Plain Declining* { *East or West* } { *Inclining* / *Reclining* } *to the Equinoctial, called an Equinoctial Decliner.*

14. 15. *A* { *South* / *North* } *Plain Declining* { *East or West* } { *Reclining* / *Inclining* } *Whereof are two sorts, the one passing above, the other beneath the Pole*

16. 17. *A* { *South* / *North* } *Plain Declining* { *East or West* } { *Inclining* / *Reclining* } *whereof are 2. sorts, the one passing above the Equinoctial, the other beneath it.*

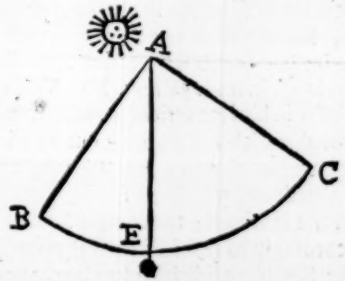
Of each of these we shall say something, but before we proceed, it will be necessary in the first place to shew how to finde the Situation of any Plain.

1. *To finde whether a Plain be Level or Horizontal.*

The performance hereof is already shewed by Mr. *Stirrup*, in his *Compleat Dyalist*, Page 58. where he hath a Scheme to this purpose, I shall onely mention it: Get a smooth Board, let it have one right edge, and near to that edge, let a hole be cut in it for a Plummet to play in; draw a Line cross the Board Perpendicular, to the streight edge thereof passing through the former hole; if then setting the Board on its smooth edge, and holding it Perpendicularly,

dicularly, ſo that the Plummet may play in the hole, if which way ſoever the Board be turned, the threed will fall (being held upright with the Plummet at the end of it, playing in the former hole) directly on the Perpendicular Line drawn croſs the Board, the ſaid Line no wayes Reclining from it, the Plain is Horizontal, otherwiſe not.

3. *To take the Altitude of the Sun without Inſtrument.*



Upon any Smooth Board, draw two Lines at right Angles, as A B and A C, and upon A as a Center with 60^{d} of a Line of Chords deſcribe the Arch B C, and into the Center thereof, at A drive in a Pin or Steel Needle, upon which hang a threed and Plummet, then when you would obſerve an Altitude, hold the Board ſo to the Sun, that the ſhaddow of the Pin or Needle may fall on the Line A C, and where the threed interſects the Arch B C, ſet a mark, ſuppoſe at E then meaſure the Ark B E on your Line of Chords, and it ſhews the Altitude ſought, for want of a Line of Chords, you may firſt divide the Arch B C into 9 parts, and the firſt of thoſe Diviſions into 10 ſmaller parts.

3. *To draw a Horizontal and Vertical Line upon a Plain.*

The readyeſt and moſt certainest way to do this, eſpecially if the Plain lean downwards from the Zenith, will be by help of a threed and Plummet held ſteadily, to make two pricks at a compe-

tent diſtance in the ſhadow of the threed on the Plain, projected by the eye, and a Line drawn through thoſe, or parallel to thoſe two points, ſhall be the Plains perpendicular or Vertical Line, and a Line drawn perpendicular to the ſaid Line, ſhall be the Plains Horizontal Line.

To finde the Reclination of a Plain.

The Reclination of a Plain, is the Angle comprehended between the Plains perpendicular, and the Axis of the Horizon, and the ſame Definition may ſerve for the Inclination, onely the upper face of a Plain, leaning from the Zenith, is ſaid to recline, and the under face to incline. In this ſenſe Mr. *Oughtred* in his Circles of Proportion, Mr. *Wells* in his Dyalling, Mr. *Newton* in his Inſtitution, and Mr. *Foſter* in his late writings, underſtand it, and ſo it is to be taken throughout this Treatiſe; but here it will not be amiſs to intimate that Mr. *Gunter*, Mr. *Wingate*, and Mr. *Foſter* in a Treatiſe of a Quadrant, publiſhed in *Anno* 1638. account the Inclination from the Horizon; the complement of the Angle here defined, and accordingly have ſuited their Proportions thereto, in which ſenſe, the Reclining ſide is ſaid to be the upper face of an Incliner; but for the future it will be inconvenient to take it any more in that Acception.

Now to finde the Reclination of a Plain, apply the ſtreight edge of the Board we uſed for trying of Horizontal Plains, to the Plains perpendicular, holding the threed and plummet ſo in the grooved hole, that it may interſect the line, parallel to the ſtreight edge of the Board, and make two pricks upon the Board where the threed paſſeth, and it will make an Angle with the ſaid Line, equal to the Plains Reclination, to be meaſured with Chords, according as was directed for taking of Altitudes; and if need require, a Line may be drawn parallel to that drawn on the Board, which repreſented the threed, which will make the ſame Angle with the Plains perpendicular, the former Line did.

The Inclination will be eaſily got, by applying the ſtreight edge of the Board, to the Plains perpendicular, and holding the threed and plummet ſo at liberty, as that it may croſs the Line parallel to the Boards ſtreight edge, and it will make an Angle therewith, equal to the Plains Inclination.

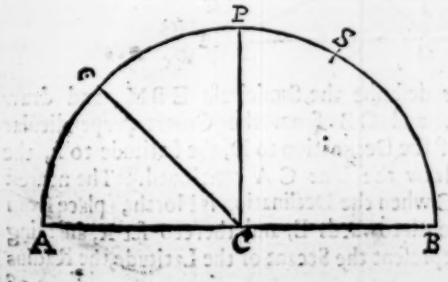
Of

Of Declining Plains.

A Plain hath its Denominations from the Scituation of its Poles: by the Poles of a Plain, is meant a Line imagined to paſs through the Plain at right Angles thereto, the Extremities of which Line on both ſides the Plain, are called its Poles. If a Plain look full South, without ſwerving, either to the Eaſt or Weſt; it is ſaid to be a direct South Plain, if it ſwerve towards Eaſt or Weſt, it is ſaid to decline thereto; if it ſtand upright without leaning, it is ſaid to be erect Now a South Plain that declines Eaſtward, the oppoſite face thereto, is ſaid to be a North Plain declining as much Weſtward, by which means the Declination of a Plain will never exceed 90^d, and to which ſenſe the following directions thoroughout this Treatiſe are ſuited. Now we may define the Declination of a Plain to be the Arch of the Horizon, contained between the true Point of Eaſt or Weſt, and the Plain, equal whereto is the Arch of the Horizon between the true North or South, and that Vertical Circle or Azimuth that paſſeth through the Plains Poles.

To finde the Declination of a Plain.

Upon a Board, having one ſtreight ſmooth edge, draw a Line parallel thereto, and thereon deſcribe a Semicircle, with the Radius of the Chords, which diſtinguiſh into two Quadrants by a perpendicular from the Center; then holding the Board parallel to the Horizon, and applying the ſtreight edge to the Wall, hold up a threed and Plummet, ſo that the ſhadow thereof may paſs through the Center, and in the ſhadow, make a mark near the Limb, from which a Line drawn into the Center ſhal repreſent the ſhadow of the threed.



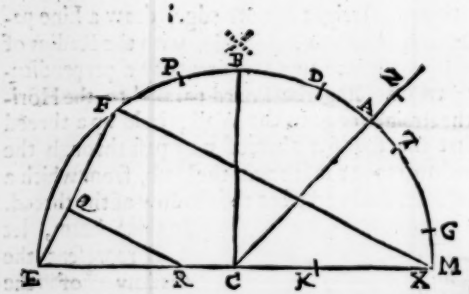
In this Scheam, let ⊙ C repreſent the ſhadow of the threed paſſing thorow the Center, at the ſame inſtant, take the Suns Altitude, and by the following directions, finde the Syns true Azi-

Azimuth. Admit the Sun be 75^d to the Eastwards of the South, prick it from ☉ to S, then the distance between P and S shews the quantity of the Plains Declination, and the Coast is also shewn, for if P fall to the Eastwards of S, the Declination is Eastwards, if to the Westwards of it, it is Westward; in this Example it is 30^d Eastwards.

We may suppose that the Reader knoweth which way from the Line of shadow to set of the Suns true Coast, that he may easily do by observing the Coast of his rising or setting, or whether his Altitude increase or decrease, and needs no directions.

A General Proportion for finding the Azimuth.

As the Cosine of the Altitude, Is to the Secant of the Latitude,
Or as the Cosine of the Latitude: Is to the Secant of the Altitude;
So is the difference of the Versed Sines of the Suns distance from the elevated Pole, and of the Ark of difference between the Latitude and Altitude.
To the Versed Sine of the Azimuth from the North in this Hemisphere.



Declination 23^d 31′ *North.*
Latitude 51^d 32′
Altitude 41^d: 34′
Azimuth 105^d *from the North*

Upon C as a Center, describe the Semicircle E B M, and draw E C M the Diameter, and C B from the Center perpendicular thereto, from B set off the Declination to D, the Latitude to L, the Altitude to A; and draw the Line C A continued. The nearest distance from D to B C when the Declination is North, place from C towards M when South towards E, and thereto set K, assuming the Diameter E M to represent the Secant of the Latitude, the Radius of

of the ſaid Secant ſhall be the Coſine of the Latitude doubled; wherefore the neareſt diſtance from L to C M doubled, ſhall be the Radius to the ſaid Secant, which prick on the Line of Altitude from C to Z, then the neareſt diſtance from Z to C M ſhall be the Coſine of the Altitude to that Radius, which extent prick from E to F, and draw the Lines E F and F M; then for the third term of the proportion, take the diſtance between A and L, and prick it from M to G, from which point take the neareſt diſtance to B C, and place it from C to X, ſo is the diſtance K X, the difference of the Verſed Sines ſought, which prick from F to Q, and draw Q R parallel to F M, ſo is R M the Verſed Sine of the Suns Azimuth from the North ſought, and C R the Sine of it to the Southwards of the Eaſt or Weſt, and B P the meaſure thereof in the Limbe, found by drawing a Line from R parallel to C B; in this Example the Suns true Coaſt is 15^{d} to the Southwards of the Eaſt or Weſt. Note alſo, that the Point R may be found without drawing the Lines E F, Q R by entring the extent K X, ſo that one foot reſting on the Diameter, the other turned about, may but juſt touch F M.

To make a Horizontal Dyal.

A Plain is then ſaid to be Horizontal, when it is parallel to the Horizontal Circle of the Sphere proper to the place of Habitation; theſe are ſuch kinde of Dyals as ſtand commonly upon poſts in a Garden.

By the former Directions, a true Meridian Line may be found, let it here be repreſented by the Line F M.

A Horizontal Dyal for the Latitude of London, *51 degrees, 32 minutes.*

Croſs the ſame with a perpendicular, to wit, the Line of Six, and draw an Oblong or ſquare figure as here is done, wherein to write the hours, from M to C ſet off ſuch a Radius as you intend to draw a Circle withall, which for convenience may be as big as the Plain will admit, and therewith upon C as a Center deſcribe a Circle, and ſet off the Radius of the ſaid Circle from F to d, from M to k, and draw the right Line F d, then prick the Poles height or Latitude of the place from f to L, and from L to S, and through the point S draw the Line M S, and it ſhall repreſent the Cock or Stiles height from

from the point L, take the neareſt diſtance to F M, and prick that extent from F to N on the Line F d, then a Ruler laid from N to k, where it interſects the Meridian is the Regulating point, or point ⊙, then from M divide the Circle into twelve equal parts for the whole hours, ſetting the Letters of the Alphabet thereto, and lay a ruler from the Regulating point to each of thoſe Diviſions, and it will interſect the Circle on the oppoſite ſide, from which Interſections, Lines drawn into the Center at M, ſhall be the hours Lines required, to be produced beyond the Center, as many as are needful, which ſhall be the hours before or after 6 in the Summer half year, the halfs and quarters are after the ſame manner to be inſcribed, by dividing each equal diviſion of the Circle into halfs and quarters.

This Circular Dyalling, was in effect, publiſhed and invented by Mr. *Foſter* in his book of a Quadrant, in *Anno* 1638. for the demonſtration of this Work will demonſtrate the truth of thoſe Circular performances, which he Operates on the back of that Quadrant, but is more expreſly hinted in his *Poſthuma*, the demonſtration whereof ſhall follow.

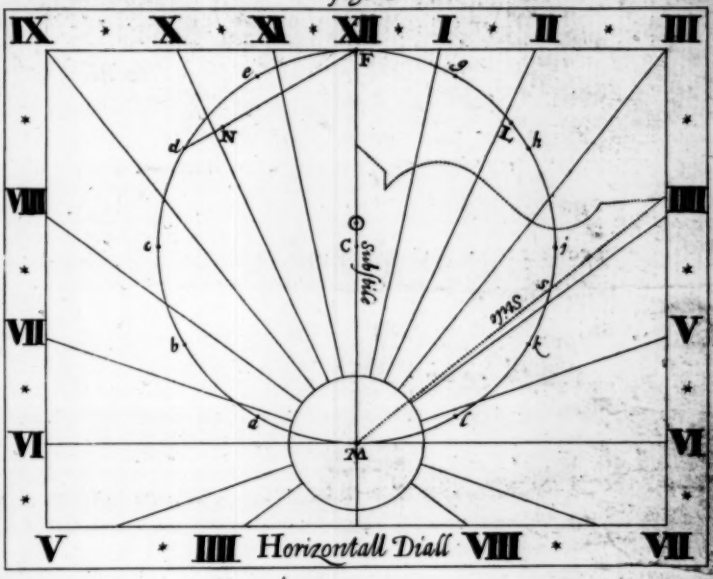
Thoſe that have plain Tables with a Frame, may have Tangent Lines put on the ſides of their Frame; and then if a Center be found upon the paper under the Frame by the Interſection of a Ruler laid over thoſe Tangents, the requiſite Diviſions of a Circle to any Radius that can be deſcribed upon the paper, will be moſt readily given without dividing any Circle or ſetting of Marks or Letters thereto, the Frame keeping faſt that paper, on which the Draught of the Dyal is made, which may alſo be ſupplyed from a Circle divided on Paſtboard cut out, which is to be Laced upon a Board over the paper whereon the Draught is to be made.

An upright full South Dyal.

This is no other then a Horizontal Dyal, in that Latitude which is equal to the complement of the Latitude of the place, you are in, onely the hours muſt not be continued beyond the Center, and the Delineation requires no other directions then the former.

On a direct North Dyal, in this Latitude, there will not be above two hours above, and two hours beneath each end of the Horizontal Line to be expreſſed, and the Stile will have the ſame Elevation and point upward.

In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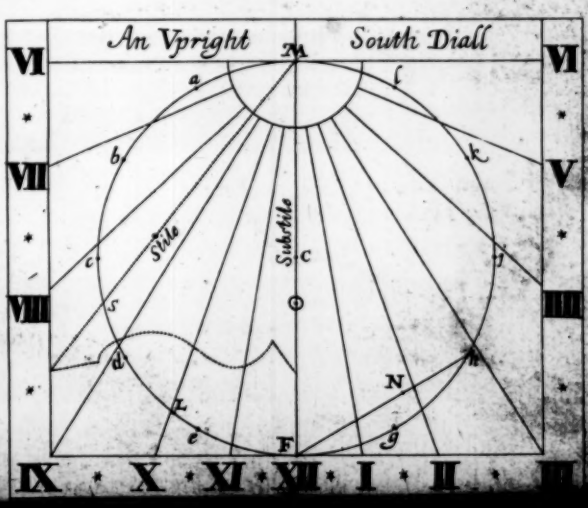
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In all upright North and South Plains, the Meridian or hour line of 12 is perpendicular to the Plains Horizontal line, if the Plain be direct the height of the Stile above the substile, is equal to the complement of the Latitude.

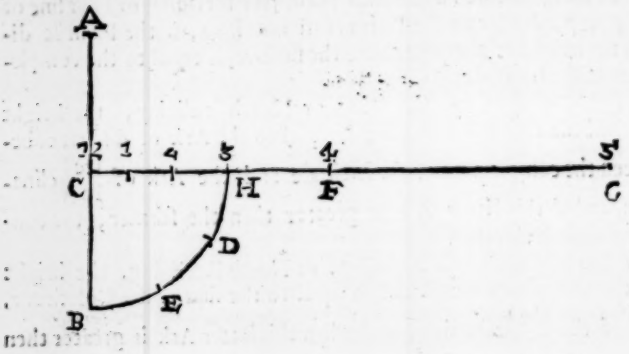
If the Plain be South Reclining, or North Inclining, the height of the Stile above the substile, is equal to the Ark of difference between the complement of the Latitude and the Ark of $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$clination, and if this latter Ark be greater then the former, the contrary Pole is elevated.

If the Plain be South Inclining, or North Reclining, the height of the Stile above the substile, is equal to the sum of the Colatitude, and of the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$clination, and when this latter Ark is greater then the Latitude, the Stiles height will be greater then a Quadrant.

Such Horizontal or direct South Dyals, both upright and leaning, whereon the stile hath but small Elevation, are to be drawn with a double Tangent Line without a Center, wherein the following directions for direct Polar plains, and those for other Oblique plains, whereon the stile hath but small elevation, will fully direct you.

All Plains that cut the Axis of the World, have a Center; but if they be parallel thereto, the stile hath no Elevation.

Such direct South Reclining, or North Inclining Plains, whose Arch of Reclination is equal to the complement of the Latitude, are parallel to the Axis, and are called Polar plains; in these the Hour Lines will be Tangent Lines of any assumed Radius, and the parallel height of the stile above the plain, must be made equal to the Radius of the Tangent, by which the hours were set off.



First draw the Tangent line CG, and perpendicular thereto ACB, upon C as a Center, with any Radius describe the Quadrant of a Circle HB, and prick off the Radius from C to A, from H to E, from B to D, from D to F, from F twice to G.

A Ruler laid from A to E findes the point 1 on the Tangent C G for the hour line of one, and laid from A to D findes the point 2 for the hour line of two, the hour line of three is at the point H, of four at the point F, of five at the point G; thus are the whole hours easily inserted.

Now to insert the halfs and quarters.

Divide the Arches B E, E D, D H into halfs, and a Ruler laid from A to those divisions wil finde points upon CH, where all the half hours under 45^{d} are to be graduated; and if a Ruler be laid from B to those respective divisions of the Quadrant B H, it will finde points on the Tangent C G, where the whole hours and halfs are to be graduated above 45^{d}, after the same manner are the quarters to be inserted; But in regard the halfs and quarters above 45^{d} will by this direction be found with much uncertainty, I have added the help following.

First, Divide the halfs and quarters under 45^{d} as now directed, then for those above make use of this Table.

In

In a direct South Polar Dyal. The distances of these hour lines.	Are equal to the distances of these hour lines.	This I do not assert to be absolutely true, but so neer the truth, that there will not arive above one thousandth part of the Radius difference in the greatest Dyal that is made, and will be more certain then any Sector, though of
3 and $3\frac{1}{4}$	$\frac{1}{4}$ and $1\frac{1}{4}$	
$3\frac{1}{2}$ and 4	$1\frac{1}{4}$ and $2\frac{1}{2}$	
$2\frac{1}{4}$ and $3\frac{1}{2}$	$1\frac{1}{2}$ doubled	
4: and $4\frac{1}{4}$	$\frac{1}{4}$ and $1\frac{1}{4}$	
$4\frac{1}{4}$ and $4\frac{1}{2}$	$3\frac{1}{2}$ and $3\frac{1}{4}$ doubled: otherwise, $1\frac{1}{4}$ and $2\frac{1}{4}$	
$4\frac{1}{4}$ and $4\frac{1}{4}$	2 and $3\frac{1}{4}$	

a Vast Radius, or then they can with convenience be prickt down the common way, by a Contingent Line, the meaning of the Table will be illustrated by one or two examples.

The distance between the hour lines of two and a quarter, and three and three quarters will be equal to twice the distance between the hour lines of twelve, and one and a half.

Also the distance between four and a quarter, and four and a half, is equal to twice the distance between three and a half, and three and three quarters; or it is equal to the distance between one and three quarters, and two and three quarters, but the former is nearer the truth.

It will be inconvenient in such plains, as also in direct East or West Dyals, to express any hour line from the substile beyond 75^{d} or 5 hours.

To these I may add some other observations which were communicated by Doctor *Richard Sterne*, to Mr. *Sutton*, which may be of use to try the truth of these kinde of Plains.

1. The distance between the hours of 3 and 4, is equal to the distance between the hours of 1 and 3.

2. The distance between the hours of 2 and 4, is double to the distance between the hours of 12 and 2.

3. The distance between the hours of 11 and 4, is double to the distance between 12 and 3, or to that between 9 and 12, and so equal to the distance between 9 and 3, also equal to the distance between 4 and 5.

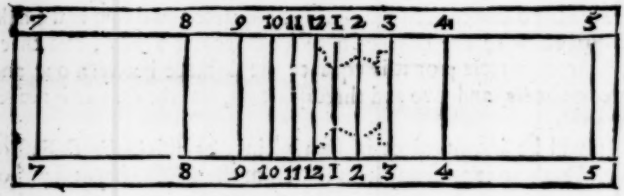
4. The distance between the hours of 11 and 5, is double to the distance between the hours of 11 and 4, as also to the distance between the hours of 4 and 5, and between 9 and 3; and quadruple to the distance between 12 and 9, or between 12 and 3.

These are absolutely true, as may be found by comparing the differences of the respective Tangents from the natural Tables.

To draw a Polar direct South Dyal.

Having drawn the Plains perpendicular in the middle of the Plain, let that be the hour line of 12, then assuming the stile to be of any convenient parallel height, that will suit the plain, making that Radius, divide a Tangent line into hours and quarters by the former directions, and prick them down on the Plain, upon a line drawn perpendicular to the Meridian or hour line of 12 on each side thereof, and through the points so prickt off, draw lines parallel to the Meridian line, and they shall be the hour lines required, as in the Example.

A Direct South Dyal Reclining 38^{d} 28$'$ *being a Polar plain in the Latitude of* 51^{d} 32$'$.



To describe an Equinoctial Dyal.

Such direct North Recliners, or South Incliners, whose $\left.\begin{matrix}\text{Re}\\\text{In}\end{matrix}\right\}$clination is equal to the Latitude, are parallel to the Equinoctial Circle, and are therefore called Equinoctial Dyals, there is no difficulty in describing of these: Divide a circle into 24 equal parts for the whole hours, & afterwards into halfs & quarters, and place the Meridian line in the Plains perpendicular, assuming as many of the former hours as the Sun can shine upon for either face, and then placing a round Wyre in the Center for the Stile, perpendicular to the Plain, and the Dyal is finished.

To

To draw a polar direct [illegible] [illegible]

[illegible]

To

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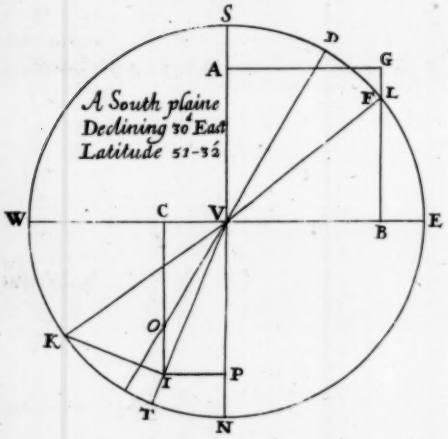

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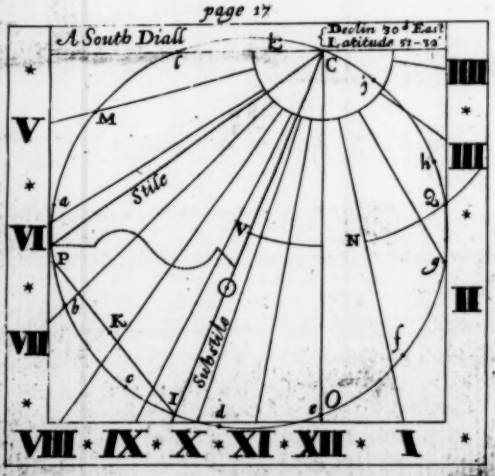


To prick off the Subſtile and Stiles height on upright Decliners in their true Coaſt and quantity.

On ſuch Plains draw a Horizontal Line, and croſs the ſame with a perpendicular or Vertical, at the interſection ſet V at the upper end of the Vertical Line ſet S, at the lower end N, at the Eaſt end of the Horizontal Line ſet E, at the Weſt end W, prick off the Declination of the Plain in its proper Coaſt from S or N to D, and draw DV through the Center, the ſame way count off the Latitude to L, and from it draw a Line into the Center; in the ſame quarter make a Geometrical ſquare of any proportion at pleaſure, ſo that two ſides thereof may be parallel to the Horizontal and Vertical Line, at the interſection of one of the ſides thereof, with the Vertical ſet A, and of the other ſide, with the Horizontal Line, ſet B, and where the Latitude Line interſects the ſide of the ſquare, ſet F.

To prick off the Subſtilar.

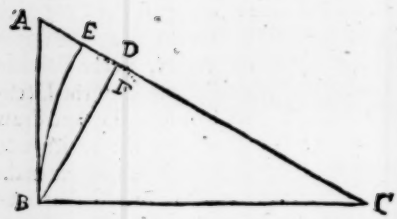
For Latitudes above 45^{d} take BF, and prick it on the Line of Declination beyond the Center from V to O, and from the point O, draw the Line OC parallel to the Vertical Line, and produced beyond O, if need require, and thereon from C to I prick the ſide of the Square, and a line drawn into the Center, ſhall be the Subſtilar line; but for Latitudes under 45^{d} prick the ſide of the ſquare from V to O, and draw OC as before, and make CI equal to AF, and a line from I drawn into the Center, ſhall be the Subſtilar line.

Stiles Height.

From the Point I erect the extent OC perpendicularly to the ſubſtilar OI, at the extremity thereof ſet K, from whence draw a line into the Center, and the Angle IVK will ſhew how much the ſtile is to be elevated above the ſubſtilar line.

In order to the Demonſtration hereof, let it be obſerved that an Angle may be prickt off by Sines or Tangents in ſtead of Chords.

To prick off an Angle by Sines or Tangents.



As in the Scheme annexed, let BC be Radius, and let there be an arch prickt off with Chords, as BE; I say, if the Tangent of the said Ark BA be taken out of a line of Tangents to the same Radius, and be erected perpendicular to the end of the Radius, as BA, a line drawn from A into the center, shall include the same Angle as was prickt off by Chords, as is evident from the definition of a Tangent.

In like manner an Angle may be prickt off by Sines, the neareſt diſtance from E to BC is the Sine of the Arch BE, ſo in like manner the neareſt diſtance from B to EC, is the Sine of the ſame Arch; wherefore if with the Sine of an Arch from the end of the Radius be deſcribed another Ark, as F, and from the Center or other extremitie of the Radius, a line drawn juſt touching the ſame, the Angle included between the ſaid line, and the Radius ſhall be an Ark equal to the Ark belonging to the ſaid Sine; and what is here done by a line of natural Sines or Tangents, by help of a Decimal line of equal parts, equal to the Radius may be done by help of the natural Tables without them.

Any Proportion relating to the ſixteen caſes of Sphœrical Triangles, amongſt which the Radius is always ingredient, may be ſo varyed, that the Radius may be in the third place, and a Tangent or a Sine in the fourth place, and then if the Ark belonging to the fourth Proportional be known, an Angle equal thereto may be prickt off by Tangents or Sines according to the nature of the fourth term, as beforh; if it be unknown, notwithſtanding an Angle equal thereto may be prickt off with Sines or Tangents according to the nature of the fourth term, from the two firſt terms of the Proportion, becauſe they are in ſuch Proportion, the firſt to the ſecond, as the Radius or third term is to the fourth; upon this Baſis follows the demonſtration of this Scheme.

The

The Demonſtration of the former Scheme for upright Decliners.

1. That the Subſtilar Line is true prickt off.

Aſſuming VB equal to VA to be Radius, then BF becomes the Cotangent of the Latitude, whereto V O is made equal, which becoming Radius, ◐C is the Coſine of the Declination, and V C the Sine. Now one of the Proportions for finding the Subſtile diſtance from the Meridian is this following:

As the Sine of the Plains Declination,
Is to the Tangent of the Latitude:
So is the Radius to the Tangent of the Subſtiles diſtance from the Horizontal Line.

From whence it follows, that if the Sine of the Declination be pricked on the Horizontal Line from the Center, as V C, and the Tangent of the Latitude, erected perpendicularly thereto, as CI equal to VA, it ſhall give a point from whence a Line drawn into the center, ſhall be the Subſtilar Line.

Now I am to prove that VA is the Tangent of the Latitude, thus it is made good: In a Tangent Line of 45ᵈ, if the Tangent of any part or ark of it be aſſumed to be Radius (as here FB the Cotangent of the Latitude equal to V O, which before was made Radius) then doth the ſaid whole Tangent line (which in this caſe is any ſide of the Square) become the Tangent of that arks complement; thus, V A becomes the Tangent of the Latitude, becauſe the Radius is a mean Proportional between the Tangent of an Ark, and the Tangent of that Arks complement; For,

As the Cotangent of the Latitude B F, *Is to the Radius* B G,
So is the Radius BF, *to the Tangent of the Latitude* BG, *there being the like Proportion between the two latter, as the two former terms.*

2. That the Stiles Height is true prickt off.

To perform this, the Coſine of the Declination O G, to that leſſer Radius, was erected perpendicularly from the point I, in the Subſtilar [illegible], and I K made equal thereto. Now one of the Proportions for calculating the Stiles height is:

As

As the Secant of the Latitude, Is to the Coſine of the Declination: So is the Radius, To the Sine of the Stiles Height.

Conſequently the Stiles height may be prickt off from the two firſt terms of the Proportion, if it can be proved that VK is the Secant of the Latitude; here let it be remembred that VP is the Tangent of the Latitude, IP the Sine of the Declination, and IK the Coſine.

By conſtruction the Angles VPI and VIK are right Angles, therefore the ſquare of VI is equal to the two ſquares of VP, and IP by 47 Prop. 1 Euclid; again the ſquare of VK is equal to the two ſquares of IK and IV, wherefore the ſquare of VK is equal to the ſum of the three ſquares of IK, IP and VP; but the ſquares of PI a Sine, and IK its Coſine are equal to the ſquare of the Radius.

But the ſquare of the Radius, and the ſquare of VP, the Tangent of the Latitude, is equal to the ſquare of VK, therefore VK is the Secant of the Latitude, becauſe the ſquare of the Radius, more the ſquare of the Tangent of any Ark, is equal to the ſquare of the Secant of the ſaid Ark.

3. That VK is equal to VF.

Let VF be an aſſumed Radius, ſo will F*B* be the Coſine of the Latitude. Again obſerve that in any line of Sines, if the Sine of any Ark be made Radius, the whole line becomes the Secant of that Arks complement, ſo here *VK* being the Secant of the Latitude is equal to VF the Radius, for let VT be made equal to VF, I ſay it holds as VO the Coſine of the Latitude, is to the Radius *VT*, ſo is *VO* the Radius to *VT* the Secant of the Latitude, which is therefore equal to *VK*.

Whence the trouble of raiſing a perpendicular in this Scheme, becauſe the ſide of the ſquare paſſeth through the complement of the Latitude, is ſhunned the point K falling in the outward circumference.

Laſtly, from all this it unavoidably follows that *VI* is the Coſine, and IK the Sine of the Stiles height to the Radius *VF*, which was to be proved.

An upright South Dyal, Declining 30 degrees Eaſtwards, Latitude 51 degrees, 32 minutes.

To draw the hour Lines of the former upright Decliner.

Firſt draw the Plains perpendicular CO, which for upright Plains

is

is the Meridian line, and with the Radius *VF* of the former Scheme, draw an occult Ark upon [redacted] as a center, and therein ſet off *NV* equal to TN of the former Scheme, and draw the line *IVC* for the ſubſtile, and with the Radius *VC* deſcribe a circle thereon.

1. *The firſt work will be to finde the Regulating Point in the Subſtile, called the Point* Sol.

Prick the Radius of the ſaid Circle from I to P, and from C to Q, and draw the Line IP, and therein make IK equal to IK in the former Scheme, then a ruler laid from K to Q, where it interſects the Subſtilar is the Regulating Point ☉.

2. *To finde a Point from whence the Circle is to be divided into* 12 *equal parts.*

Lay a Ruler from O to the point ☉, and it interſects the Circle on the Oppoſite ſide, at it ſet M.

3. *To divide the Circle.*

From the point M before found, lay a ruler through the Center V, and it will finde a Point on the other ſide the circle, at it ſet F, the points M F divide the Circle into halfs, and each half is to be divided into ſix parts, the Radius VM will eaſily divide each half into three parts, and then it will be eaſie to divide each of thoſe parts into halfs, and ſo the whole Circle will be divided into 12 equal parts, and if it be deſired to inſcribe the half hours and quarters, then muſt each of thoſe parts be divided into halfs and quarters.

Having thus divided the Circle into 12 parts, diſtinguiſh them with the Letters of the Alphabet.

4. *To draw the hour lines, and to number them.*

Lay a Ruler to each of thoſe Letter diviſions, and upon the Regulating Point ☉ in the Subſtile, and it will finde points on the Oppoſite ſide of the Circle from which if lines be drawn into the Center at C they ſhall be the hour lines required.

5. *An inconvenience ſhunned.*

When the points through which the hour lines are to be drawn, fall neer the Center, the hour lines cannot be drawn from the ſaid points with certainty.

In this case let any hour line near the said point be produced, if need require, and upon C as a Center with the Radius C V describe the Ark of a Circle from the produced hour-line, take half the distance between the point, where the produced hour line, cuts the Circle, and the point through which the hour line proposed is to be drawn, and prick that extent off in the Arch swept, setting one foot where the Arch swept cuts the hour line already drawn, from whence half the distance was measured, and the other foot will finde a Point in the said Arch, through which the hour line desired is to pass: Thus we inscribed the hour line of three in the former Dyal.

When the North Pole is elevated, the Center of the Dyal must be below, but when the South Pole is elevated, as in this example, it must be above, and the Point ⊙ must be alwayes found in the other part of the Diameter most remote from the Center.

6. The Stiles height may be transferred from the former Scheme into this, by pricking the Arch TK twice in this Circle, and it findes a point, from whence a line drawn into the Center, shall represent the Stile.

That the hour lines are true Delineated.

The Point Sol found in the Substile, divides the Diameter of the Circle drawn on the said Substile in such Proportion, as the Radius is to the Sine of the Stiles height, by reason of the equiangled Triangles, whose Bases bear such proportion as their perpendiculars.

The next work performed by dividing the Circle into 12 equal parts, and finding points on the opposite side, by laying a ruler over those equal parts through the point Sol, carrieth on this Proportion.

As the Radius is to the Tangent of the hour from the Substile,
So is the Sine of the Stiles height
To the Tangent of the hour line, from the Substile, being the proportion used in all Dyals, whereby to set off the hours.

In the third figure following, let B A represent the Substile, and let the Regulating Point be at M, so that B M bears such Proportion to MA, as the Radius doth to the Sine of the Stiles height, let the perpendicular BHE represent a line of Tangents, whose Radius is equal to B C, and the perpendicular AD, another line of Tangents

to

to the ſame Radius both infinitely produced, and let BE repreſent the Tangent of ſome particular Arch or hour to be drawn, from E through the point M draw a right line to D, and the Proportion lies evident in right lines.

As the Radius B M: *Is to the Tangent of the hour line from the Subtile* B E:: *So is the Sine of the Stiles height* M A *To the tangent of the hour line from the ſubſtile* A D.

If from every degree of a Tangent line, lines be drawn into the Center of the Circle proper to that Tangent, they ſhall divide that Quadrant to which they belong into 90 equal parts, this follows from the definition of Tangents, but if the lines be drawn from every degree of the ſaid tangent to the extreamity of the diamer, they ſhall divide a Semicircle into 90 equal parts by 20 Prop. 3. Euclid. becauſe an Angle in the Circumference, is but half ſo much as it is in the Center.

Thus the Semicircle BGA is ſuppoſed to be divided into 90 equal parts from the Tangent BE, by lines drawn to A, as is the line EGA, ſo is the other Semicircle from the other Tangent AD, by lines drawn to B, as is BFD; Now if it can be demonſtrated that the points GMF are in a right line, it follows that the ſame Proportions may be carried on from the equal diviſions of theſe Semicircles, as were done in the right Tangent lines BE and AD, this Propoſition being very well Geometrically demonſtrated by my loving friend Mr. *Thomas Harvy*; his Demonſtration thereof ſhall hereafter follow.

Having found the diſtance of the hour line, from the ſubſtile A B, if B be made the Center of the Dyal, the right line BF drawn into the Center, ſhall repreſent the hour line propoſed, making the ſame Angle with the Subſtile, as was found by the proportion; note that no hour can be further diſtant on one ſide or other of the Subſtile then 90^{d}, and the ſaid Ark of diſtance will be found in one of the Semicircles, and if the Center were not placed in the Circumference, the Angle found would be extended beyond its due quantity, in all upright Decliners or leaning Plains: The firſt Proportion carried on, is this;

As the Sine of the Stiles height:
Is to the tangent of the diſtance between the Meridian and ſubſtile::
So is the Radius: to the tangent of the inclination of Meridians::

Whereby is found the point from whence the Circle is to begin to be divided into 12 parts for the whole hours with their halfs and

quarters, and thoſe equal diviſions give the Angles between the Meridian of the plain, and the reſpective hours, called by ſome the Angles at the Pole, and then the work is the ſame as before: Now to the Propoſition.

Conſtruction.

The right lines AD, BE are parallel, and touch the Cirele in the extreams of the Diameter AB, DE is drawn at pleaſure, cutting the Tangents in D and E, and the Diameter in M, BD and AE cutteth the Circumference in F and G, it is required to be proved that the Points F, M, and G, are in a right line.

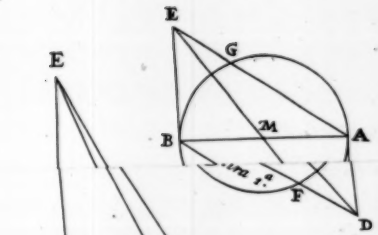
Draw the lines MF, MG, firſt if AD be = BE then AM is = MB, for the Triangles ADM and BEM are equiangled, therefore M is the Center. Again AD being = BE and the Angle *DAB* = *EBA* & *AB* common to both Triangles *DAB*, *EBA* therefore the Angles *DBA*, *EAB* are equal, and the double of them *FMA*, *GMB* the Angles at the Center, are alſo equal, wherefore *FM* and *MG* is one and the ſame right line which was to be proved.

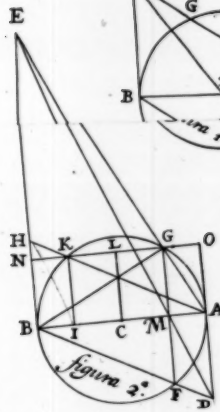
Secondly, If *AD* be not equal to *BE*, as in the ſecond, third, & fourth Scheams let *BE* be greater, and make *BH* = *AD* and *BI* = *AM*, and draw the right line *AH* cutting the Circumferenee in K; draw likewiſe *IK*, *IH*, *GK*, and extend *GK* infinitely both wayes, which ſhall be either parallel to *AB* (as in the ſecond figure) or cut it in the point P on one ſide or the other, as in the third and fourth figures, in each from the points A and B, and the Center C at right Angles to GK, draw the lines *AO*, *BN*, *CL* laſtly draw *BG*, from which Conſtruction it willfollow, That

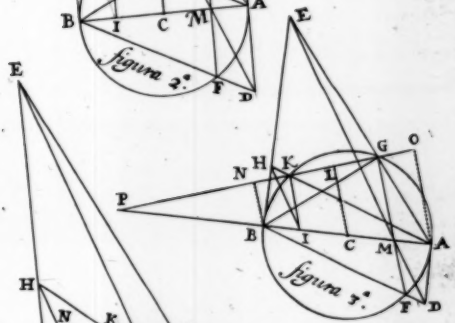
1. *HI* is parallel to *ED* { for *HB* is = *AD* and *BI* = *AM* and the Angles *HBI* and *DAM* are equal, therefore the angle *AMD* = *BME* is = *BIH* wherefore *HI* is parallel to *ED*.

2. *IK* is parallel to *MF* { for *HB* is = *AD* and the Angle *HBA* = *DAB*, and *AB* common to both the Triangles *HBA* and *DAB*, therefore the Angles *HAB* and *DBA* are equal, wherefore *AK* = *BF*, but *AK* being = *BF* and *IA* = *MB*, and the Angle *KAI* = *MBF*, therefore *KI* is parallel to *MF*.

3. The


E
G
B
M
A
figura 1.ª
F
D


E
H
N
K
L
G
O
B
I
C
M
A
figura 2.ª
F
D


E
N
H
K
G
O
P
B
I
C
M
A
figura 3.ª
F
D

E
H
N
K
L
G
O
P
B
I
C
M
A
figura 4.ª
F
D

3. The Triangles A H E and A G K, are equiangled, ſo alſo are the Triangles A H B and A G O, becauſe the Angle A K G = A B G is = A E B and E A H common to both the Triangles A H E and A G K, therefore they are equiangled; again the Angles A H B and and A G O being equal, the right Angled Triangles A B H and A O G are alſo equiangled.

4. N L is equal to L O, becauſe B C is equal to C A and N K is = G O, becauſe K L is = L G; now in the equiangled Triangles A B H, A O G as B H : H A :: O G : G A and in the Triangles A H E, A G K as H A : H E :: G A to G K, therefore *(ex æquo)* it will be as B H : H E :: O G : G K, but as B H : H E :: B I : I M for I H is parallel to E M, therefore as B I : I M :: O G : G K but N K is = O G, therefore B I : I M :: N K : K G, but I C is half of I M and K L half of K G, therefore B I : I C :: N K : K L, therefore by Compoſition of Proportion, as B C : I C :: N L : K L, alternately, as B C : N L :: I C : K L.

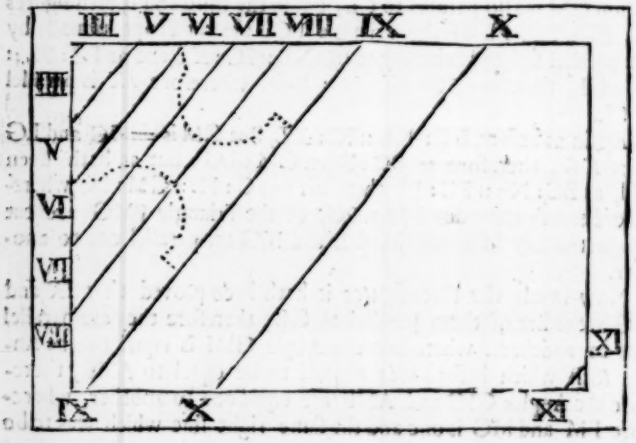
In the ſecond figure B C is = N L, therefore I C is = K L, wherefore (becauſe I C is alſo parallel to K L) I K is parallel to C L, by like reaſon G M is alſo parallel to C L, but in the third and fourth figures the ſides P C, P L of the Triangle P C L are cut Proportionally by the parallel B N, and as before, as B C : N L :: I C : *K L*, and as P C : P L :: B C : N L, therefore P C : P L :: I C : *K L*, wherefore *I K* is parallel to C L.

Again as before, B C : N L :: I C : *K L*, But C M is = I C and L G is = *K L*, therefore as B C : N L :: C M : L G, but as hath been ſaid, as B C : N L :: P C : P L, therefore as P C : P L :: C M : L G, wherefore becauſe the ſides P M : P G, of the Triangle P M G are cut proportionally in C and L, C L and M G are parallel one to another.

Now in all the three figures it hath been proved that *I K* and M G are either of them parallel to C L, therefore they are parallel one to another, wherefore the Angle G M I is equal to the Angle *K* I B which before was proved to be equal to A M F, therefore the Angles G M I and A M F are equal one to another, wherefore F M and M G is one and the ſame right line which was to be proved.

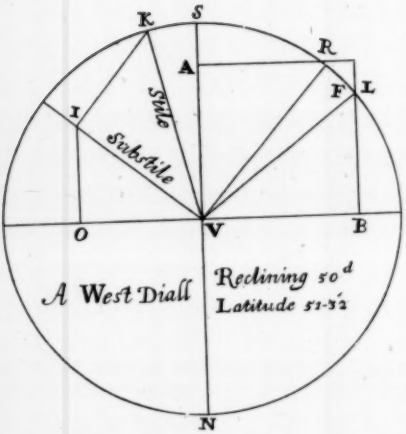
To draw an East or West Dyal.

Let the hour line of six, which is also the Substilar line, make an Angle equal to the Latitude of the place, with the Plains Horizontal line, above that end of it that points to the Coast of the elevated Pole, then draw a line perpendicular to the Substilar line, which some call a Contingent line, and with such a Radius as you determine the Stile shall have parallel height above the Substile: divide a Tangent line of hours and quarters according to the direction for direct Polar Recliners, then through those divisions draw lines parallel to the Substile and they shall be the hour lines required; thus the hours of 5 and 7, are each of them Tangents of 15^d from six, and so for the rest, let them be numbred on each side of six (being the Substile) for an East dyal with the morning hours, for a West dyal with the afternoon hours, the one being the complement of the other to 12 hours, and therefore we have but one example, namely an East dyal for the Latitude of *London*.

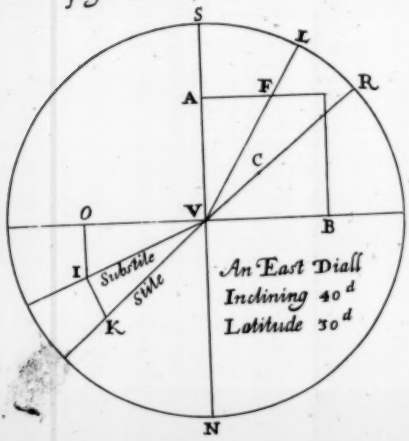


How to fill this or any other Plain, with any determined number of hours, shall afterwards be handled.

To


K
S
R
A
F
L
I
Stile
Subtile
O
V
B
A West Diall
Reclining 50 d
Latitude 51-32
N

page 27


S
L
F
R
A
C
O
V
B
I
Subtile
Stile
K
An East Diall
Inclining 40 d
Latitude 30 d
N

To prick off the requisites of an East or West Reclining or Inclining Dyal in their true Scituation and quantity.

In these Plains, first draw the Plains perpendicular, and cross it with a Horizontal line, which is also the Meridian line; in any one of the quarters, make a Geometrical square, as before directed for upright decliners, and from the Plains Vertical in the said quarter, count off the Latitude to L, and from the Horizontal line in the said quarter count off the Reclination or Inclination to R, and from L and R, draw lines into the Center, and where the Latitude line intersects the side of the square, set F.

1. *To prick off the Substile.*

For Latitudes above 45^{d} place BF from V the Center in the Horizontal line for Recliners Northwards, for Incliners Southwards, and thereto set O, and through the said point, draw a line parallel to the Vertical, and place the neareſt distance from A to RV on the former parallel, from O for Recliners upwards to I, but for Incliners downwards, and a line thence drawn into the Center, shall be the Substile.

2. *The Stiles height.*

Place the neareſt distance from *B* to RV on a perdendicular raised from the point I in the Substile, and it findes the Point K, whence a line drawn into the Center, shall represent the Stile. In the other Hemisphere the words Northwards and Southwards, must be mutually changed.

For Latitudes under 45^{d}

The side of the square must be placed from V to O and AF must be placed on the line of Reclination or Inclination from V to C, the neareſt distance from C to VA placed on the perpendicular, passing through O for Recliners upwards, but for Incliners downwards, findes the Point I, through which the Substile is to pass, and the neareſt distance from C to *VB* raised perpendicularly on the point I in the Substile, findes the point K for the Stile, as before.

Demonstration. 1. *For the Substile.*

In these Plains *VB* or *VA* being Radius *FB* is the Cotangent of the

the Latitude and the neareſt diſtance from A to RV, is the Coſine of Reclination or Inclination to the ſame Radius, and the neareſt diſtance from *B* to RV, the Sine thereof; this for Latitudes above 45, but for leſſer Latitudes, AF the Tangent of the Latitude was made Radius, and thereupon the Radius or ſide of the ſquare bebecame the Cotangent of the Latitude, and the neareſt diſtance from C to VA was the Coſine, and from C to *VB* the Sine of the Reclination or Inclination, ſo that the preſcribed Conſtruction in both caſes erects the Coſine of the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$clination on the Cotangent of the Latitude, which is made good from this proportion.

As the Cotangent of the Latitude

Is to the Coſine of the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$*clination from the Zenith*

So is the Radius: To the tangent of the Subſtilar from the Meridian.

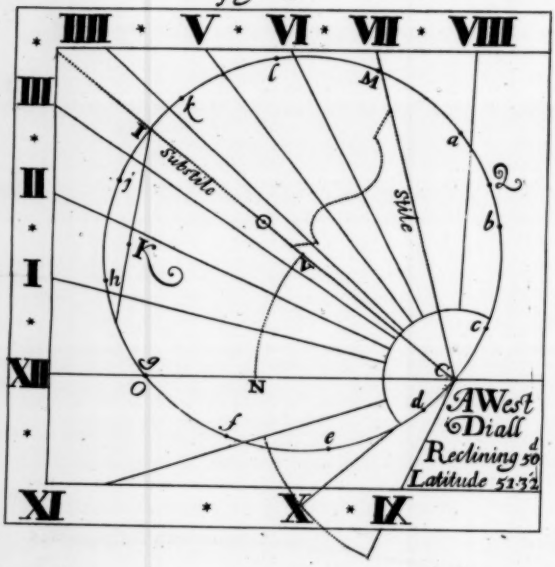
2. *For the Stile.*

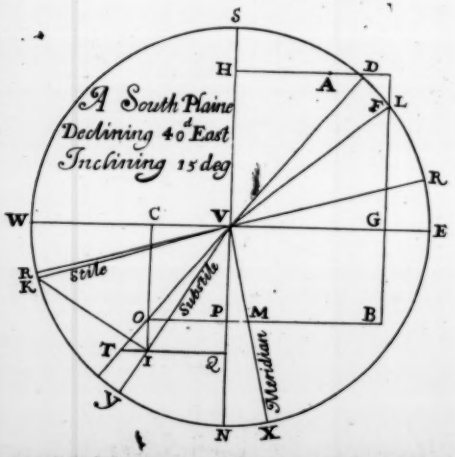
A proportion that will ſerve to prick it off, is,

As the Coſecant of the Latitude, is to the Sine of the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$*clination,*

So is the Radius: To the Sine of the Stiles height.

Admit the Stiles height were calculated, and truely prickt off by Chords, I ſay if the Sine of the Latitude be made a Radius, as the ſide of the ſquare is in Latitudes above 45^d and as AF is in Latitudes under 45^d the Radius to the ſaid Sine will become the Coſecant of the Latitude; now we ſhall prove that VK is the Coſecant of the Latitude, the three ſquares IK, IO, and OV, are equal to the ſquare of VK, as before aſſerted, two of theſe ſquares IK and IO are equal to the ſquare of the Radius, the third ſquare, *viz.* of VO is the ſquare of the Cotangent of the Latitude, but the ſquare of the Radius, more the ſquare of the Cotangent of the Latitude is equal to the ſquare of the Coſecant of the Latitude; I ſay then, if from K a perpendicular be let fall to VI, ſo as that it may be equal to the Coſine of the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$clination ſhall prick off the Stile true by the directions for pricking off an Angle by Sines, whereto the proportion for the Stiles height is ſuited and would paſs through the point I, by reaſon


IIII
V
VI
VII
VIII
III
II
I
XII
XI
X
IX
Substile
Stile
A West Diall
Reclining 50d
Latitude 51.32


A South Plaine
Declining 40d East
Inclining 15 deg
Stile
Substile
Meridian

reaſon of the equality of the three ſquares to the ſquare of *VK*, then by conſequence the contrary follows, having the point I firſt given, the Point K may be found according to the preſcribed Conſtruction.

A Weſt Plain, Reclining 50 degrees, Latitude 51 degrees, 32 minutes.

The Point Sol, Subſtile, Stile, and hour-lines, are all found after the ſame manner as in an upright Decliner, the Subſtile being ſet off from the Meridian line here, after the ſame manner as it was from the Meridian there.

A South Plain Declining Eaſt 40 degrees, Inclining 15 degrees, Latitude, 51 degrees, 32 minutes.

To prick off the Requiſites in all Declining, Reclining, and Incliing Plains in their true Coaſt and quantity.

Now followeth thoſe Directions inlarged, which as I ſaid in the Epiſtle, I received from Mr. *Thomas Rice.*

UPon any Plain firſt draw a true Horizontal line, at the Eaſt end thereof ſet E, and at the Weſt end W, croſs the ſaid line with a perpendicular, which may be called the Plains perpendicular, by ſome termed the Plains Vertical line, or line of Reclination, at the interſection of theſe two lines ſet V, and upon it as a Center deſcribe a Circle (the Radius whereof may be equal to 60^d of a line of Chords) at the upper end of the Vertical line ſet S, and at the lower end N.

As the Declination is, ſet it off with Chords from S or N towards the true Coaſt, and at it ſet D, from whence draw a line through the Center; alſo ſet off the Latitude of the place the ſame way and in the ſame quarter, and at it ſet L, from which draw a line to the Center.

From that end of the Horizontal line, towards which the declination was counted, ſet the Inclination (which as well as the Reclination is reckoned from the Zenith, the former being the Denomination of the under, the latter of the upper face) upwards towards S, and the Reclination downwards towards N, and at it ſet R, from whence draw a line through the Center to the other ſide of the Circle.

In the same quarter of Declination, draw HA parallel to the Horizontal line, and FG parallel to the Vertical line, in a Geometrical square, of like and of any convenient distance from the Center at pleasure, and where the Latitude line intersects the side of the square, let the letter F be placed.

On the line of Declination beyond the Center, make VO equal to FG, and draw OB parallel to the Horizontal line, continued till it meet with the side of the square FG produced, at the point of concurrence, set B, and where it intersects the Plains perpendicular, set P, and draw OC parallel to the Vertical line cutting WE at C, and make HA equal to OC or BG, now by help of the three points A, B, C thus found, the requisites will be easily prickt off.

1. *The Substilar line.*

The neareſt distance from A to RV, set on the line CO (produced if need be) from C to I the same way the distance was taken from A, that is, if downward or upward, the other must be so too, will shew where VI the Substilar Line is to be drawn.

2. *The Stiles height.*

The least distance from B to R V, set on a perpendicular raised upon the Substile from the point I will finde the point K, from whence draw a Line into the center at V, and the Angle IVK will shew how much the Stile is elevated above the Substile, and if the work be true, VK and VF will be equal, whence it follows, that the trouble of raising the mentioned perpendicular may be shunned.

3. *The Meridian line.*

The least distance from C to RV, set upon the Line OPB, from P on that side, which is farthest from the Line RV, will finde the Point *M*, from whence a Line drawn into the Center, shall be the meridian Line.

And I adde that on all North Recliners in the Northern Hemisphere, the meridian Line must be drawn through the center on the other side, and then the construction of the Scheam will place it below the Plains Horizontal Line, which is its proper Scituation for the said upper face, and for the under face the Scheam placeth it true without caution.

4. *A*

4. *A Polar Plain how known.*

If the line RV fall juſt into B, the Plain is a Polar Plain, in ſuch a Plain the Stile hath no height, but is parallel to the Axis, in this caſe the Inclination of Meridians muſt be known, directions for ſuch Plains muſt afterwards follow.

But if the line RV fall between the Points B and P, then muſt the Subſtile Stile and Meridian be all drawn through the Center, and ſtand beyond on the other ſide.

Annotations on the former Scheam.

1. That for Latitudes under 45^{d} this conſtruction of the Scheam, ſuppoſeth the ſides of the ſquare produced, which will therefore be lyable to large excurſions or other inconveniences, wherefore for ſuch Latitudes, I ſhall ſomewhat vary from the conſtruction preſcribed.

2. In finding the Subſtilar line, in ſtead of erecting CI upon VC, you may prick the ſame on the Vertical line VN, and thereto erect VC, and get the point I poſſibly with more certainty by finding the interſection of two Arks where the ſaid Point is to paſs.

3. In pricking off the Meridian line, the diſtance of C from the Center may be doubled or tripled, but ſo muſt likewiſe VP, and the neareſt diſtance from C to RV erected on a line drawn parallel to WE, paſſing through the Point P ſo found, and in ſtead of drawing ſuch a parallel, the Point M may be found by the interſection of two Arks.

4. That this Scheam placeth the requiſites of all Dyals in their true coaſt and quantity, yet notwithſtanding if this Scheam be held before a Looking-Glaſs, the Effigies thereof in the Glaſs ſhews how the Scheam would happen and place the Requiſites, namely, the Stile, Subſtile and Meridian, for a Plain of the ſame Denomination, but declining to the contrary Coaſt.

And if the face of the ſaid Scheam be laid upon a Window, and the Subſtile, Stile and Meridian be continued through the Center on the backſide thereof, it ſhews you how theſe requiſites are to be placed on the oppoſite ſide of the Plain, which being done, may be held before a Looking-Glaſs as before, and will be repreſented for the contrary Declination of that oppoſite face: the truth of all which will be confirmed from the Scheam it ſelf.

This Scheam for Declining, Reclining, or Inclining Plains, uſeth a new method of Calculation, derived from an Oblique Triangle in the Sphere, wherein there is two ſides with the Angle comprehended, given to finde both the other Angles, which is reduced by a perpendicular to two right Angled Triangles, from which the following proportions are derived.

I ſhall therefore firſt deliver the ſaid method, then demonſtrate that the ſaid proportions are carried on in the Scheam; and laſtly from the Sphere, ſhew how thoſe proportions do ariſe.

1. To finde a Polar Plains Reclination or Inclination.

As the Radius, Is to the Coſine of the Plains Declination,
So is the Cotangent of the Latitude, To the Tangent of the Reclination or Inclination ſought.

2. To finde the diſtance of the Subſtile or Meridian line, from the plains perpendicular for a Polar plain.

As the Radius, Is to the Sine of a Polar Plains Reclination,
So is the Tangent of the Declination, To the Tangent of the Subſtilar line from the Plains perpendicular.

3. The Inclination of Meridians.

As the Radius, Is to the Sine of the Latitude,
So is the Tangent of the Declination, To the Tangent of the Inclination of Meridians.

Affections of a Polar Plain.

The Subſtilar on the upper face, lies above that end of the Horizontal line, towards the Coaſt of Declination, and the Meridian lyes parallel to the Subſtile beyond it, towards that end of the Horizontal line that is towards the Coaſt of Declination.

For Declining, Reclining, or Inclining Plains.

Firſt finde a Polar Plains Reclination for the ſame Declination. Then for South Recliners and North Incliners, get the difference, but for North Recliners and South Incliners, the ſum of a Polar plains Reclination, and of the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$clination of the plain propoſed, and then it holds.

1. For the Subſtile.

As the Coſine of the ſaid Ark of difference or ſum according as the Plain leans Northward or Southwards,

Is to the Sine of the Polar Plains Reclination,
So is the Tangent of the Declination,
To the Tangent of the Subſtilar from the Plains perpendicular.

2. For the Stiles height.

As the Radius, Is to the Coſine of the Subſtiles diſtance from the Plains perpendicular,
So is the Tangent of the ſum or difference of Reclinations, as before limited, To the Tangent of the Stiles height.

3. Meridians diſtance from the plains perpendicular.

As the Radius, Is to the Sine of the $\left.\begin{matrix} Re \\ In \end{matrix}\right\}$ *clination,*
So is the Tangent of the Declination,
To the Tangent of the Meridian from the Plains perpendicular.

4. Inclination of Meridians.

As the Sine of the Stiles height,
Is to the tangent of the diſtance between the Meridian and Subſtile,
So is the Radius, to the tangent of the Inclination of Meridians.

For South Recliners or North Incliners, the difference between the ſubſtiles diſtance from the plains perpendicular, and the Meridians diſtance therefrom, is equal to the diſtance between the Meridian and Subſtile; the like for ſuch North Recliners or South Incliners, as Recline or Incline more then an Equinoctial Plain, having the ſame declination. but if they lean above it, or have a leſſer Reclination, the ſum is the diſtance between the Meridian and the Subſtile.

The three firſt Proportions, beſides the finding of a Polar Reclination, are uſed in the Scheam for the placing of the Requiſites, and the latter proportion in the Circular Scheam for drawing the hours.

Another proportion for finding the Inclination of Meridians by Calculation, is:

As the Coſine of the Latitude,
Is to the ſine of the ſubſtiles diſtance from the Plains perpendicular,
So is the Coſine of the $\left.\begin{matrix} Re \\ In \end{matrix}\right\}$ *clination of the Plain,*
To the ſine of the Inclination of Meridians.

The

The Reclination of an Equinoctial Plain to any assigned Declination, is necessary for the determining of divers affections: The Proportion to finde it, is:

As the Radius: Is to the Cosine of the Plains Declination:
So is the Tangent of the Latitude,
To the Tangent of the Reclination sought.

The upper face of an Equinoctial Plain, is called a North Recliner, the Meridian descends from the end of the Horizontal line opposite to the Coast of Declination, the Substilar line is the hour-line of six, and maketh right Angles with the Meridian line.

Directions for the true Scituating of the Meridian and Substile suited to the former method of Calculation.

1. *For Plains leaning Northwards.*

If a South Plain recline more then a Polar Plain, having the same Declination, the Plain passeth beneath the Pole of the World, the North Pole is elevated upon the upper face, the Substile and Meridian line lye above that end of the Plains Horizontal line, towards the Coast of Declination, the Substilar line being next the Plains perpendicular.

For the under face being a North Incliner, the South Pole is elevated, the lines lye in the same position below the plains Horizontal line, and on the contrary side of the plains perpendicular.

If a South Plains Reclination be less then the Polar Plains Reclination, the Plain passeth above the Pole, and the North Pole is elevated on the under face, being the inclining side.

The Substile and meridian lye above that end of the Plains Horizontal line that is opposite to the Coast of Declination, the meridian being neareſt the Plains perpendicular, for the upper face being a South Recliner, the South Pole is elevated, and the lines lye in the same Position below the Plains horizontal line, but on the contrary side of the plains perpendicular descending below that end of the Horizontal line, opposite to the Coast of Declination.

2. *For Plains leaning Southwards generally on the upper face the North Pole is elevated, on the under face the South Pole.*

To place the Substile.

Such North Recliners whose Reclination is less then the complement

ment of a Polar plains Reclination, the Subſtile is elevated above the end of the Horizontal line contrary to the Coaſt of Declination, and on the under face, being a South Incliner, the Subſtilar is depreſſed below the end of the Horizontal line, oppoſite to the Coaſt of Declination.

But when the Reclination is more then the complement of the Reclination of a Polar plain, the Subſtile is to lye below the plains Horizontal Line, from that end oppoſite to the Coaſt of Declination.

But for South Incliners, being the under face, the Subſtile is elevated above the end of the Horizontal line, oppoſite to the Coaſt of Declination.

To place the Meridian.

On all North Recliners the Meridian lies below the Horizontal Line from that end thereof, oppoſite to the Coaſt of Declination, becauſe at noon the Sun being South, caſts the ſhadow of the Stile to the Northwards.

On the under face, being a South Incliner, it muſt always be placed below the Horizontal Line, below that end of it toward the Coaſt of Declination.

Theſe Directions ſuppoſe the Declination to be denominated from the Scituation of that face of the plain on which the Dyal is to be made, and the Horizontal line for all Dyals that have Centers, is ſuppoſed to paſs through the ſame.

Now to the Demonſtration of the former Scheam.

1. 'Tis aſſerted that if RV fall into the point B, the plain is a Polar plain, in which caſe the Stile is parallel to the Axis of the world.

Demonſtration.

Every Declining plain may have ſuch a Reclination found thereto, as ſhall make the ſaid plain become a Polar plain, and the proportion to finde it, may be thus:

As the Tangent of the Latitude, Is to the Coſine of the Declination, So is the Radius, To the Tangent of the Reclination ſought.

In the former Scheam, if we make FG the Cotangent of the Latitude

titude Radius, the ſide of the ſquare will be the Tangent of the Latitude, now VO equal to FG, being Radius, OC equal to GB, is the Coſine of the Declination; wherefore a Line drawn into the Center from B, ſhall include the Angle of a Polar plains Reclination agreeable to the two firſt terms of the proportion, and to the directions for pricking off an Angle by Tangents.

2. *That the ſubſtile is true prickt off.*

Upon V as a Center with the Radius VB, imagine or deſcribe a Circle then is BG equal to VP, the Sine of a polar plains Reclination, which is equal to HA, and the Ark comprehended between A and B, will be a Quadrant.

But in a Quadrant any line being drawn from the Limbe paſſing through the Center, the neareſt diſtance from the end of one of the Radij will be the Sine of the Ark thence counted, and the neareſt diſtance from the other Radius thence counted, will be the Sine of the former Arks complement; ſo in this Scheam the neareſt diſtance from B to RV, when a plain Reclines, is the Sine of the Ark of difference, but when it inclines of the ſum of the Reclination of the Plain propoſed, and of the Reclination of a Polar plain, and the neareſt diſtance from A to RV, is the Coſine of the ſaid Ark.

Make VQ on the plains perpendicular equal to IC, which is equal to the neareſt diſtance from A to RV, and from the point Q erect the perpendicular QT, whereto the line PO will be parallel, and conſequently there will be a proportion wrought: Thus it lyes,

As VQ the Coſine of the ſum of the Polar plains Reclination, and of the Reclination of the plain propoſed,

Is to PV equal to BG, the Sine of a Polar Plains Reclination.

Theſe two terms are of one Radius, namely VB,

So is the tangent of the Declination QT, *To the tangent of the ſubſtiles diſtance from the plains perpendicular* PO,

Theſe two terms are to another Radius, namely QV, then if the Tangent of an Ark be erected on its own Radius, as here is Q I equal to CV, and a line be drawn from the extreamity into the Center, the Angle belonging to that Tangent ſhall be prickt off agreeable to the general direction.

3. *That the ſtiles height is true prickt off.*

The Proportion altered to bring the Radius in the third place will be,

As

As the Secant of the ſubſtiles diſtance from the Plains perpendicular,
Is to the tangent of the ſum or difference of Reclinations, as before limited,
So is the Radius, To the tangent of the ſtiles height.

In the Scheam making VQ Radius, VI becomes the Secant of the ſubſtiles diſtance from the Plains perpendicular, and the nearſtdieſtance from B to RV, is the tangent of the difference or ſum of the Reclinations, which when VB was Radius, was but the Sine thereof, the reaſon why it now becomes a tangent, is becauſe the Coſine of the ſaid Ark VQ, is made Radius: But,

As the Coſine of any Ark, Is to the Radius,
So is the Sine of the ſaid Ark, To the tangent of the ſaid Ark.

Therefore the neareſt diſtance from B to RV equal to IK, being erected thereon, and a line from the extreamity drawn into the Center, ſhall prick off the ſtiles height ſuitable to the two firſt terms of the former proportion, and to the general direction for pricking off an Angle by Tangents.

4. That the Meridian is true prickt off, the Proportion to effect it: Is,

As the Cotangent of the Plains Declination,
Is to the Sine of the Re / In *clination,*
So is the Radius, To the tangent of the Meridian from the Plains perpendicular.

If VC be made Radius, then is CO equal to VP the Tangent of the complement of the Plains Declination, and the neareſt diſtance from C to RV, is the Sine of the Re / In clination to the ſame Radius, which is erected perpendicularly on VP ſuitable to the two firſt terms of the proportion and the General Direction.

Laſtly, that VK is equal to VF, theſe Symbols are uſed, q ſignifieth ſquare, + for more or Addition, = equal.

VBq = VQq + IKq, the reaſon is becauſe VB being Radius, VQ is the Coſine of an Ark to that Radius, and IK the Sine by conſtruction.

VBq = VGq + GBq, therefore these two squares are equal to the two before.

If to the latter part of each of these Equations, we adde QIq or rather its equal POq, the sum shall be equal to VKq.

I say then VQq + IKq + POq = VKq, this will be granted from the former Demonstration for upright Decliners.

Again,

VGq + GBq + POq = VFq, therefore VF is equal to VK, this cannot be denyed, because, VGq + FGq = VFq.

And the two squares GBq, or rather VPq + POq are equal to GFq, which is equal to VOq by Construction.

To draw the hour-lines for the former South Plain, Declining Eastwards 40 *degrees, Inclining* 15 *degrees Latitude,* 51 *degrees,* 32 *minutes.*

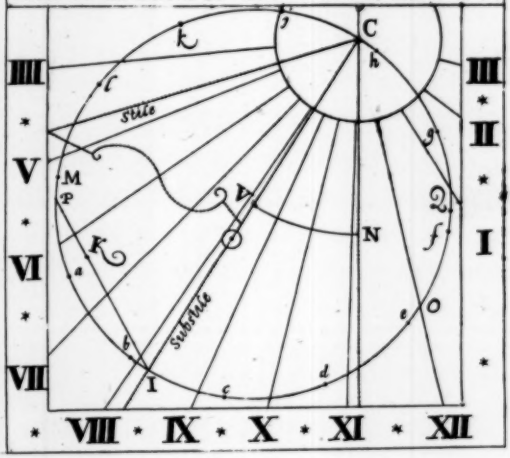
First having assigned the Center of the Dyal, through the same draw the plains perpendicular, represented by the prickt line CN, and with the Radius of the former Scheam upon C as a Center, describe an Occult Ark, and therein set off NV equal to the substiles distance from the plains perpendicular, and through the point V draw the Substilar, and upon V as a Center, describe the Circle, and prick off the Meridians distance from the Substile in the former Scheam, namely YX twice in this Circle from I to O, and draw CO for the Meridian, after the same manner set off the Stile, then finde the Regulating point Sol, divide the Circle, and draw the hour-lines according to former Directions, and when hour-lines are to lye both above and below the Center, they are to be drawn through.

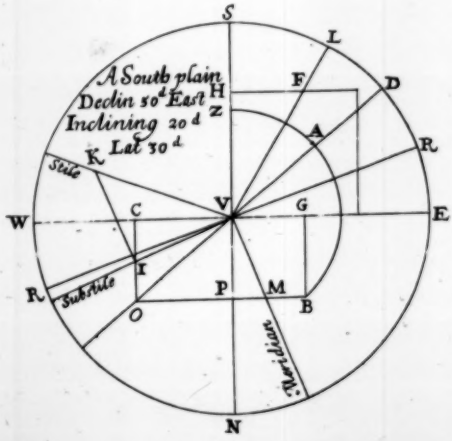
To fit the Dyalling Scheam, for Latitudes under 45 degrees.

A South Plain Declining 50 *degrees East, Inclining* 20 *degrees, Latitude* 30 *degrees.*

The former Construction would serve, if the sides of the square were produced far enough, but to shun any such excursion, make VO equal to the side of the square, and through the point F, draw a parallel to the Plains perpendicular, and where the parallel OP produced intersects it, is the point B, upon V as a Center, with the extent

A South Diall Declin 40d East Inclin 15d Latit 51-32





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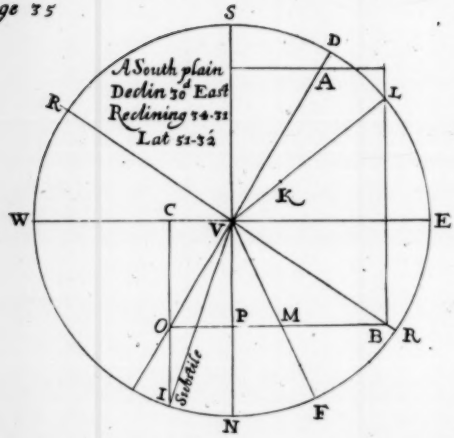

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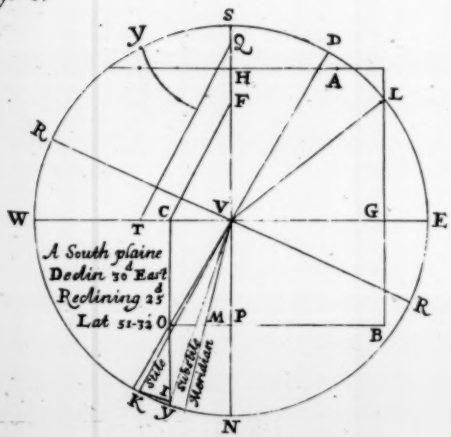


tent VB, draw the Arch ZB, and prick off a Quadrant thereof from the point B, and it will finde the point A, the point C is found no otherwise then before; and now having these three points, the whole work is to be finished according to those Directions, to which when the Stile hath a competent height, nothing need more be added, unless it be some examples.

How to draw such Dyals whereon the Stile hath no elevation as Polar Plains, or but very small elevation, as in upright far Decliners, and many leaning Plains.

These plains are known easily, for if the Re/In clination pass through the point R, the plain is a Polar plain, and the Substile is to be found by the former Construction, which the Scheam makes the same with the Meridian; moreover, another Ark is to be found, called the Inclination of Meridians: The proportion to finde it, is:

As the Radius, Is to the Sine of the Latitude,
So is the tangent of the Declination,
To the tangent of the Inclination of Meridians.

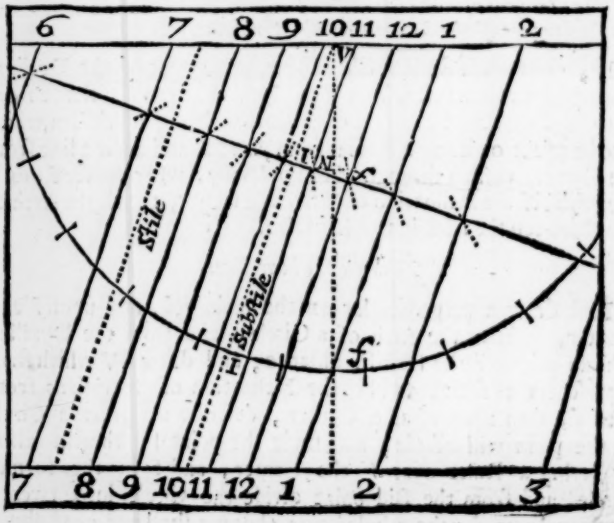
If CO be made Radius, CV will be the Tangent of the Declination, which enter on VL from V to K, and the nearest distance from K to *SV* shall be the Tangent of the Inclination of Meridians, which is to be prickt on its own Radius from P to M, and draw a line from the Center, passing through M to the Limbe, whereto set F, so is the Arch NF the Inclination of Meridians sought, to wit, the Arch of time between the substile and Meridian.

To draw the hour-lines.

First draw a perpendicular on the Plain VN, and upon *V* as a Center, describe the Arch of a Circle, and from the Dyalling Scheam prick off the substiles distance, and draw IV which shall represent the same, as also the Inclination of Meridians from I to f, then upon *V* as a Center, describe as great a Circle as the plain will admit, and finde the point f, therein also, by laying a Ruler over *V* the Center, and f in the former Circle, and from the said point divide the Circle into twenty four equal parts for the whole hours (but we shall not need above

half of them) then determine what ſhall be the parallel height of the Stile above the Subſtile, and prick the ſame on the Subſtilar line, from *V* the Center to I, through which point draw a Contingent line at right Angles to the Subſtile, and laying a Ruler from the Center over each of the diviſions of the Circle, through the points where it interſects the Contingent line, if lines be drawn parallel to the Subſtile, they ſhall be the hour-lines required, the hour-line that belongs to the point f being the Meridian-line or hour-line of 12, after the ſame manner are the halfs and quarters to be inſcribed, if the Contingent line be too high, the Center *V* may be placed lower, if it be required, to fit ſo many hours preciſe to the Plain; firſt draw it very large upon ſome Floor, and then it may be proportioned out for a leſſer Plain at pleaſure, as was mentioned for Eaſt or Weſt Plains.

A South Plain, Declining Eaſtwards, 30 degrees, Reclining 34 degrees, 31 minutes.



The

The Scheam placeth all things right for Equinoctial Reclining Plains, without any further caution.

To draw the Hour-lines on ſuch Plains, where the Stile hath but ſmall elevation.

A South Plain Declining 30 *degrees Eaſtwards, Reclining* 25 *degrees, Latitude* 51 *degrees* 32 *minutes.*

In theſe Plains, becauſe the hour-lines will run cloſe together, the Dyal muſt be drawn without a Center, by help of two Contingent lines, and firſt of all the Inclination of Meridians muſt be known, thereby is meant the Arch of time between the Subſtile and the Meridian line or hour line of 12, and that may be found ſeveral ways; here I ſhall follow the proportion in Sines before delivered.

Making *VI* Radius, prick the ſame on the Latitude line VL, and from the point found, take the neareſt diſtance to the Horizontal line, place this extent from *V* to F, and draw CF; then take the neareſt diſtance from R to the Plains Vertical S N, which place from *V* to Q, then draw QT parallel to FC, ſo is VT the Sine of the Inclination of Meridians, which may be eaſily meaſured in the Limbe by the Arch SY, by drawing a line parallel to the Plains perpendicular from the point T, or by pricking the ſame on HA produced, and laying a Ruler thereto, or by drawing the touch of an Arch with VT upon S as a Center, then a Ruler laid from *V* touching the outward Extremity of that Arch, findes the point Y.

Moreover, we need not make *VI* Radius, but prick the neareſt diſtance from L to VE, from V upwards, then the neareſt diſtance to the Plains perpendicular from the Interſection of the Subſtile, with the Limbe muſt be placed from *V* towards W, and from the two points thus found, a line drawn, and the reſt of the work, as before.

To draw the Hour-lines.

Firſt draw the Plains perpendicular GN, and draw an Occult Arch, wherein prick down NY and NK, and draw the Subſtile and Stile as before, making the ſame Angles.

Through any two points in the Subſtile, as at A and B, draw two right lines continued, making right Angles therewith.

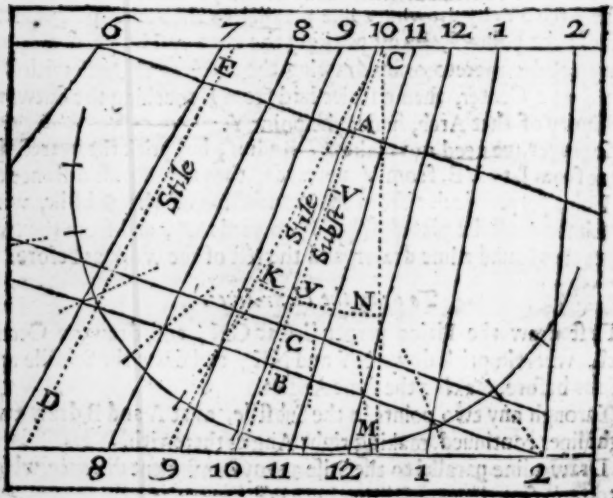
Draw a line parallel to the Stile at any convenient diſtance, which is to repreſent the new Stile, as here DE.

Take

Take the neareſt diſtance from B to DE, and ſet it on the Subſtile from B to *V*, alſo the neareſt diſtance from A to DE, and ſet it from *V* to C, through which point draw another line perpendicular to the Subſtile.

Upon *V* as a Center, deſcribe the Arch of a Circle of as large a Radius as the plain will admit, and from the ſubſtile on the ſame ſide thereof the Meridian happened in the former Scheam, ſet off the Inclination of Meridians, and it findes the point M, from whence divide the Circle into 24 equal parts, and draw lines from the Center V through thoſe parts, cutting both the Contingent lines B and C, the reſpective diviſions of the Contingent line C, muſt be transferred into the Contingent line A, and there be made of the like diſtance from the ſubſtile as in the ſaid line C, then lines drawn through the Diviſions of the two Contingent lines A and B, ſhall be the reſpective hour-lines required.

A South Plain Declining Eaſtwards 30 *degrees, Reclining* 25 *degrees, Latitude* 51 *degrees,* 32 *minutes.*



After

After the ſame manner muſt ſuch Eaſt and Weſt Recliners or Incliners, that have ſmall elevation of the Stile, and upright or decliners be pricked down, and in theſe plains the Meridian many times muſt be left out, the proportion to finde the Inclination of Meridians for upright Decliners: Is,

As the Radius, Is to the Sine of the Latitude,
So is the Cotangent of the Plains Declination,
To the Cotangent of the Inclination of Meridians.

In the Dyalling Scheam, making CV Radius, CO is the Cotangent of the Declination, which enter on the Latitude line VL, and take the neareſt diſtance to SV, which extent prickt upon CO, and it findes a Point, through which a line drawn from the Center to the Limbe, ſhall ſhew the inclination of Meridians to be meaſured from N.

Otherwiſe:

That we may not transfer large Diviſions on the Contingent line from a ſmall Circle, and that the Plain may be filled with any determined number of houres, ſuch as by after Directions ſhall be found meet, draw any right line on a Board or Floor that ſhall repreſent the Plains perpendicular, as CN, and from the ſame ſet off the Subſtile and Stiles height from the general Scheam as before, drawing a line parallel to the Stile as DE, alſo a line perpendicular to the Subſtile, which I call the Floor Contingent, and that it may be large, let it be of a good diſtance from the Center; from the point of Interſection at Y, take the neareſt diſtance to the parallel Stile, which prick from Y to *V*, and upon *V* as a Center deſcribe as large a Circle as may be with convenience, and from the Subſtile ſet off the Inclination of Meridians therein to M (which Ark refers to *V* as its Center) and from the ſaid Point divide it into 24 equal hours (or fewer, no more then are required) and laying a Ruler over *V*, and thoſe reſpective diviſions graduate them on the large Contingent line, I ſay from this large Contingent line thus drawn and divided, we may proportion out the Diviſions of two (or many) Contingent lines that are leſſer, and thereby fill the Plain with any proper number of hours required.

Then in Order to drawing the hour-lines on the Plain.

The

The firſt work wil be to draw the larger contingent line on the plain, which may be drawn any where at pleaſure: for performing whereof, note, that what Angle the Subſtile makes with the Plains perpendiculer, the Contingent line is to make the ſame with the Horizontal line, and the complement thereof, with the *Vertical* line; alſo draw another Contingent line above this, parallel thereto at any convenient diſtance.

In the bigger Contingent line aſſume any two points to limit the outward moſt hours that are intended to be drawn on the Plain, as admit on the former Plain, I would bring on hours from ſix in the morning, to two in the afternoon, between the ſpace A and B of the greater Contingent, take the ſaid extent AB, and upon the point B at 2 of the Floor Contingent, deſcribe an Ark therewith, to wit, L, then from A draw the line AL, juſt touching the outward extreamity of the ſaid Ark.

I ſay the neareſt diſtance from D to AL, being pricked on the Plains greater Contingent from A to D, findes a point therein through which the Stile is to paſs.

Alſo the neareſt diſtance from Y to AL, findes the ſpace AY on the Plains greater Contingent, and through the point Y a line drawn perpendicular to AB, ſhall be the Subſtilar line.

The Stile is to be drawn through the Point D, making an Angle with the Plains Contingent line equal to the complement of its height above the Subſtile in this example 80^{d} $57'$, to wit, the Arch NY.

The reſpective neareſt diſtances to AL, from each hour-point, in the Floor Contingent, being pricked on the Plains greater Contingent from A towards B, findes Points therein, through which the hour-lines are to paſs.

The next work will be to limit one of the extream hours on the Plains leſſer Contingent, and that muſt be done by proportion.

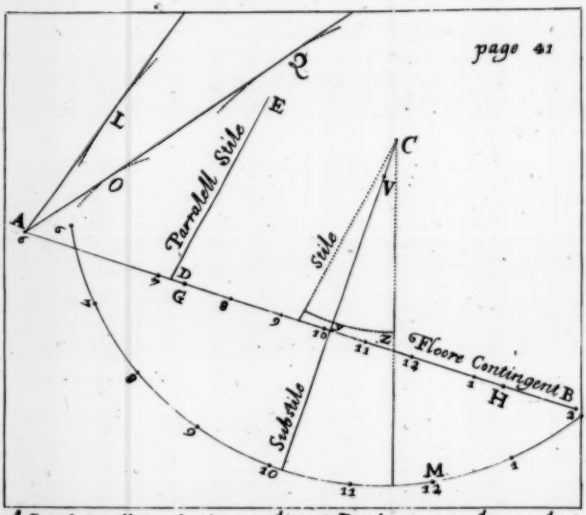
As the Diſtance between the parallel ſtile and ſubſtile on the Floor Contingent,

Is to the diſtance between the ſtile and ſubſtile on the Plains leſſer Contingent,

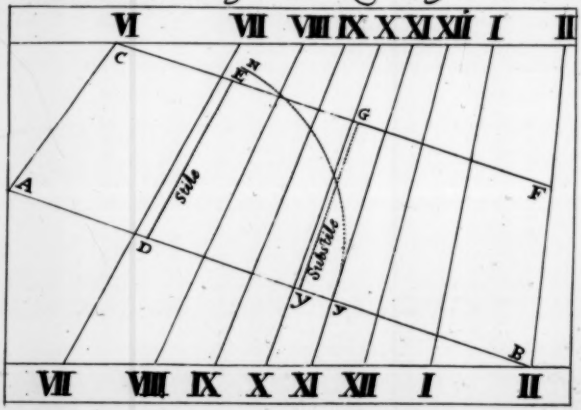
So is the diſtance between the ſubſtile and either of the extreams or outward hours on the Floor Contingent,

To the diſtance between the ſubſtile, and the ſaid outward hour on the Plains leſſer Contingent.

This



A South Diall Declining 30d East Reclining 25d Lat 51-32



This proportion is to be carryed on in the Draught on the Floor, place DY from A to G on the Floor Contingent, and with the extent EG taken from the Plains leſſer Contingent, upon G on the Floor Contingent, draw the Arch O, and from A draw a line touching the outward extreamity of the ſaid Arch, and let it be produced.

The neareſt diſtance from Y to AO, being prickt on the Plains leſſer Contingent, reaches from G to C, the point limiting the outward hour of ſix.

Then if the neareſt diſtances to the line AO, be taken from all the reſpective hours on the Floor Contingent, and placed on the Plains leſſer Contingent from C towards F, you will finde all the hours points required, through which and the like points on the Plains greater Contingent, the hour-lines are to be drawn.

Here note, that the extent DY on the Floor Contingent, may be doubled or tripled; if it be tripled it reacheth to H, alſo the extent EG on the plains leſſer Contingent, is to be encreaſed after the ſame manner, and an Ark therewith deſcribed on H before found, as Q, and by this means the line AO will be drawn and produced with more certainty, then by the Ark O near the Center.

And what is here done by help of the Stiles diſtance from the Subſtile, may be done by help of the outward hour, if the diſtance of the ſaid hour-line from the Subſtile be found Geometrically or by Calculation, for the ſaid hour-line will make an Angle with the Plains Contingent lines, equal to the complement of the Ark of its diſtance from the Subſtile.

This Plain is capable of more hours which cannot conveniently be brought on.

After the ſame manner are upright far Decliners to be dealt with-all, and all other Plains having ſmall height of Stile.

But to limit the outward hours on Polar plains, and Eaſt or Weſt Plains, the trouble will not be half ſo much.

A South Plain Declining Eaſtwards 30 *degrees, Reclining* 25 *degrees, Latitude* 51 *degrees,* 32 *minutes.*

A ſecond method of Calculation for Oblique Plains.

By the former method the Meridians diſtance from the Plains perpendicular is to be found, and the Polar Reclination Calculated.

Then for South Recliners, or North Incliners, get the difference, but for North Recliners or South Incliners the sum, of the Polar Plains Reclination, and of the Reclination of the Plain proposed, and it holds.

As the Cosine of the Polar Plains Reclination,
Is to the Sine of the former sum or difference,
So is the sine of the Latitude, To the Sine of the stiles height.

Which Pole is elevated is easily determined, by comparing the Reclination of the proposed Plain with the Polar Reclination, and all other affections are to be determined, as in the first Method.

Then for the Substile, and Inclination of Meridians.

As the Cosine of the stiles height,
Is to the Sine of the Plains Declination,
So is the Cosine of the Latitude,
To the Sine of the substiles distance from the Plains perpendicular:
And so is the Cosine of the Reclination,
To the Sine of the Inclination of Meridians.

This method ariseth from the aforementioned Oblique Triangle in the Sphere, in which by help of two sides, and the Angle comprehended, the third side is first found, and the other Requisites by the proportions for Opposite sides and Angles.

Proportions for upright Decliners.

1. To finde the Substiles distance from the Meridian.

As the Radius, Is to the Sine of the Declination,
So is the Cotangent of the Latitude,
To the Tangent of the Substile from the Meridian.

2. Angle of 12, and 6.

As the Radius, Is to the Sine of the Plains Declination,
So is the Tangent of the Latitude,
To the Tangent of the Angle between the Horizontal line and six.

3. Inclination of Meridians.

As the Sine of the Latitude, Is to the Radius,

So

So is the Tangent of the Declination,
To the Tangent of the Inclination of Meridians:

4. Stiles height.

As the Radius, Is to the Cosine of the Latitude,
So is the Cosine of the Plains Declination,
To the Sine of the stiles height.

These Arks are largely defined in my Treatise, *The Sector on a Quadrant.*

For East and West Re / In *cliners,*

The complement of the Latitude of the place, is such a new Latitude: in which they shall stand as upright Plains, and the complement of their Re / In clination is their new Declination in that new Latitude, having thus made them upright Decliners, the former proportions will serve to Calculate all the Requisites.

In all upright Plains, the Meridian lyeth in the plains perpendicular, and if they Decline from the South (in this Hemisphere) it is to descend or run downward; if from the North it ascends, and the Substile lyeth on that side thereof opposite to the Coast of Declination.

In East or West Re / In cliners, it lyeth in the plains horizontal line, on the Inclining side the South Pole is elevated, but on the upper side the North Pole, and the Substile lyeth above or below that end of the Meridian line, which points to the Pole elevated above the Plain.

On all plains whatsoever to Calculate the hour distances.

As the Radius, Is to the Sine of the stiles height above the substile,
So is the tangent of the Angle at the Pole,
To the tangent of the hour-lines distance from the substilar line.

By the Angle at the Pole, is meant the Ark of difference between the Ark called the Inclination of Meridians, and the distance of any hour from the Meridian for all hours on the same side of the Meridian the Substile falls, and the sum of these two Arks for all hours on the other side the Meridian.

All hours on any Plain go to the contrary Coaſt of their Scituation in the Sphere, thus all the morning or Eaſtern hours, go to the Weſtern Coaſt of the plain, and all the evening or Weſtern hours, go to the Eaſtern Coaſt of the Plain.

A third Method of Calculation for leaning Plains, that is for all ſorts of Plains that do both Decline, and alſo Incline or Recline.

They may be referred to a new Latitude, in which they ſhall ſtand as upright Plains, and then they will have a new Declination in that new Latitude; which two things being found, the former Proportions for upright Decliners will ſerve to Calculate all the Arks required.

How this may be done on a Globe, is not difficult to apprehend, having ſet the Globe to your Latitude, let one of the Meridians of the Ecliptick or Longitude in the heavens, repreſent a Declining Reclining Plain, this Circle interſects the Meridian of the place in two Points, the one above, the other beneath the Horizon: Imagine the Globe to be ſo fixed, that it cannot move upon its Poles, then elevate or depreſs the Globe ſo in the Meridian that the point of Interſection above the Horizon may come under the Zenith, then will the Pole of the world be elevated above the Horizon to the new Latitude ſought, and where the Meridian of Longitude that repreſents the Plain interſects the Horizon it ſhews the new Declination.

Or it may be thus apprehended: The diſtance between the Pole of the world, and that point of Interſection that repreſents the Zenith of the new Latitude, is the complement of the ſaid new Latitude, and the diſtance between that point, and the Equinoctial is the new Latitude it ſelf; the new Declination is the complement of the Angle between the plain and the meridian of the place, an Ark uſually found in Calculation under this denomination.

To finde theſe Arks by Calculation.

As the Radius, Is to the Coſine of the Plains Declination,
So is the Cotangent of the $\left.\begin{matrix} Re \\ In \end{matrix}\right\}$ *clination from the Zenith,*
To the tangent of the Meridional Ark, namely the Ark of the Meridian between the Plain and the Horizon.

And

And this is the first thing Master *Gunter* and others finde; for South Recliners / North Incliners } the one being the upper, the other the under face, get the difference between this Ark and the Latitude of the place, the complement of the said residue, remainder, or difference, is the new Latitude sought; but for North Recliners or South Inclines, the difference between this fourth Arch and the complement of the old Latitude is the new Latitude.

To finde the new Declination:

As the Radius, Is to the Cosine of the Re / In } *clination,*
So is the sine of the old Declination, To the sine of the new.

This method is hinted to us in Mr. *Fosters Posthuma*, also in his Book of Dyalling in *Anno* 1638, where he refers leaning plains to such a Latitude wherein they may become East or West Recliners, but that method is to be deserted, as multiplying more proportions then this, and doth not afford that instrumental ease for pricking down the hours that this doth.

Affections determined.

Such South Recliners, whose meridional Arch is less then the Latitude, pass beneath the Pole, and have the North-Pole elevated above them, but if the meridional Ark be greater then the Latitude, they pass above the Pole, the North Pole is elevated on the under face, all other affections are before determined. If the meridional Arch be equal to the Latitude, the plain is a Polar plain; for plains leaning Southwards, if the meridional Arch be equal to the complement of the Latitude, the plain is an Equinoctial plain, if it be more, the plain hath less Reclination then an Equinoctial plain, if it be less it hath more, and all affections necessary for placing (and Calculating) the meridian line were before determined.

This method of Calculation findes the Substiles distance from the meridian, not from the plains perpetdicular, wherefore it must be shewed how to place it in Plains leaning Southwards, for plains leaning Northwards use the former directions.

To

To place the Subſtile in North Recliners.

In theſe plains the Meridian and Subſtilar are to meet at the Center, and not being drawn through, will make ſometimes an Acute, ſometimes an obtuſe Angle.

When the Plains Meridional Ark is greater then the Colatitude, they make an Obtuſe Angle, in this Caſe, having firſt placed the Meridian line, above it prick off the complement of the diſtance of the Subſtile from Meridian to a Semicircle.

But when the Meridional Ark is leſs then the Colatitude, prick off the ſaid diſtance it ſelf above the Meridian line.

In South Incliners.

When the Plains Meridional Ark is greater then the Colatitude, the Subſtile and Meridian make an Acute Angle, when it is equal to the Colatitude, they make a right Angle, when it is leſs then the Colatitude they make an Obtuſe Angle, and muſt be prickt off by the complement of their diſtance to a Semicircle, the Subſtile always lying on that ſide of the Meridian, oppoſite to the Coaſt of Declination.

A fourth Method of Calculation for leaning Plains.

An Advertiſement.

In this method of Calculation for all Plains leaning Northward, both upper and under ſide their Declination is the Arch of the Horizon between the North and the Azimuth of the plains South Pole, ſo that their Declination is always greater then a Quadrant; But for all Plains leaning Southwards, both upper and under face, their Declination is the Arch of the Horizon between the North and the Plains North Pole, wherefore it is always leſs then a Quadrant; in this ſenſe Declination is uſed in the following Proportions.

As the Sine of half the ſum of the complements, both of the Latitude and of the Reclination, Is to the Sine of half their difference,
So is the Cotangent of half the Declination,
To the Tangent of a fourth Arch.

Again,

As the Coſine of half the ſum of the former Complements,
Is to the Coſine of half their difference, So

So is the Cotangent of half the Declination,
To the tangent of a seventh Arch.

Get the sum and difference of the fourth and seventh Arch, then if the Colatitude be greater then the complement of the Reclination, the sum is the Substiles distance from the plains perpendicular, and the difference the Inclination of Meridians.

But if it be less, the difference is the Substiles distance from the Plains perpendicular, and the sum the Inclinations of Meridians.

To place the Substile.

For Plains leaning Southwards, when the Angle of the Substile from the Plains perpendicul is less then a Quadrant, it will on the upper face lye above that end of the Horizontal line that is opposite to the Coast of Declination, and on the under face lye beneath it, but when it is greater, it will lye below the said end, on the upper face, and above it on the under face, but this will not be till the Reclination be more then the complement of the Reclination of a Polar plain that hath the same Declination; for plains leaning Northwards the Directions of the first Method suffice.

To place the Meridian.

Either Calculate it, and place it according to the directions of the first and second Method, or else Calculate it by this Proportion.

As the Radius, Is to the Sine of the Stiles height,
So is the tangent of the Inclination of Meridians (when it is Obtuse, take its complement to a Semicircle)
To the tangent of the Meridian line from the substile.

For Plains leaning Northward, the first directions must serve, but for Southern Plains the second, because the distance of the Meridian is Calculated from the Substile supposed to be placed, and here the work is converse to that, for in that we supposed the Meridian placed, and not the Substile.

For the Stiles height.

As the Sine of the fourth Arch, Is to the Sine of the seventh Arch,
So is the tangent of half the difference of the complements both of the Latitude and Reclination,
To the tangent of an Arch sought.

How

How much the ſaid Ark being doubled wants or exceeds 90ᵈ, is the Stiles height.

In South Recliners, if the ſaid Ark being doubled, is leſs then 90ᵈ, its complement is the elevation of the North Pole, and the Plain falls below the Pole.

But if the ſaid Arch exceed 90ᵈ the Plain paſſeth above the Pole, and the exceſs is the elevation of the North Pole on the under face of a South Recliner, called a North Incliner, and the affections were determined in the firſt method where the Declination hath its Denomination from that Coaſt of the Meridian to which the Plain looketh.

Theſe methods of Calculation may not preciſely agree one with another; though all true, unleſs the parts Proportional be exactly Calculated from large Tables in every Operation, which to do as to the Examples in this Book, my leiſure would not permit; This laſt method is derived alſo from the former Oblique Triangle, the Proportions here applyed, being demonſtrated in *Trigonometria Brittanica* by Mr. *Newton*.

The Demonſtration of the former Proportions

In projecting the Sphere, it is frequently required to draw an Arch through any two different Points within a Circle, that ſhall divide the ſaid Circle into two equal Semicircles

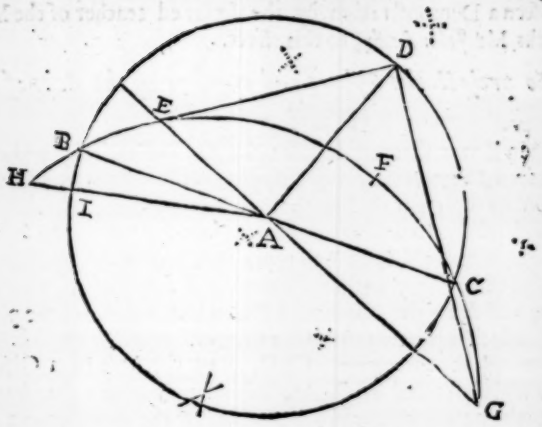
Conſtruction.

1. Draw a line from one of the given points through the Center, for conveniency through that point which is moſt remote.
2. From the Center raiſe a Radius perpendicular to that line.
3. And from the ſaid point draw a line to the end of the Radius.
4. From the end of the Radius raiſe a line perpendicular to the line laſt drawn, and where it interſects the former line drawn through the firſt point and Center, is a third point given, deſcribe a Circle through theſe three Points, and the Propoſition will be effected.

Example.

Let it be required to draw the Arch of a Circle through the two points E and F that ſhall divide the Circle BD into two equal parts.

Opera-



Operation.

From E draw EG, through the Center A, make AD perpendicular thereto, joyn ED, and make DG, perpendicular to ED cutting EG in G, through E, F, and G, draw the Arch of a Circle which will divide the Circumference BDC into two equal parts in B and C, that is, if CA be drawn, it will pass through B, if not, let it pass above or below, as let it pass below and cut BFE in H.

Demonstration.

By construction EDG and DAG are right Angles; therefore □ AD = ▭ EAG by 13 Prop. 6 Euclid. because EDG being a right Angle, AD is a mean Proportional between EA and AG, but ▭ EAG should be = to ▭ CAH by 35. Prop. of 3 Euclid. therefore ▭ CAH = □ AD.

But □ AD = □ AI that is = ▭ GAI, therefore ▭ CAH = ▭ CAI which is absurd, therefore CI cannot pass below B, the same absurdity will follow if it be thought to pass above it, therefore CA produced, will fall in the point B, wherefore BDC is a Semicircle, which was to be proved: And hereof I acknowledge I

have ſeen a Demonſtration by the Learned teacher of the Mathematicks, Mr. *John Leak*, to this effect.

To project the Sphere and meaſure off the Arks of an upright Decliner.

Upon Z as a Center, deſcribe the Arch of a Circle, and croſs it with two Diameters at right Angles in the Center, whereto ſet NESW to repreſent the North, Eaſt, South and Weſt.

Prick off the Latitude from N to L, and lay a Ruler to it from E, and where it cuts NZ, ſet *P* to repreſent the pole.

Prick off the Declination of the plain from E to A, and from S to D, and draw the Diameter AZB, which repreſents the plain, and DZC, which repreſents the Poles thereof.

Through the three points CPD, draw the Arch of a Circle, and there will be framed a right Angled Triangle ZHP right Angled at H, in which there will be given the ſide *ZP* the complement of the Latitude, with the Angle *PZH* the complement of the Declination.

Whereby may be found the Stiles height repreſented by the ſide *PH*, the Subſtiles diſtance from the plains perpendicular repreſented by *ZH*, and the Angle between that Meridian which makes right Angles with the plain, and the Meridian of the place repreſented by the Angle *ZPH*, ſhewing the Arch of Time between the Subſtile and meridian, called the Inclination of Meridians, from which Triangle are educed thoſe proportions delivered for upright Decliners.

To meaſure off theſe Arks. 1. *The Subſtile.*

A Ruler laid from *D* to H, findes the point F in the Limbe, and the Arch CF is the meaſure of the Subſtiles diſtance from the meridian, to wit, $21^d\ 41'$.

2. *The Stiles height.*

Set off a Quadrant from F to G, lay a Ruler from G to *D*, and where it interſects *BZ*, ſet ⊙ which is the Pole of the Circle *CPD*, lay a Ruler from ⊙ to *P*, and it interſects the Limbe at I, ſo is the Arch AI the meaſure of the Stiles height, to wit, $32^d\ 32'$.

3. *Inclination of Meridian.*

Lay a Ruler from *P* to ⊙, and it interſects the Limbe at T, and the

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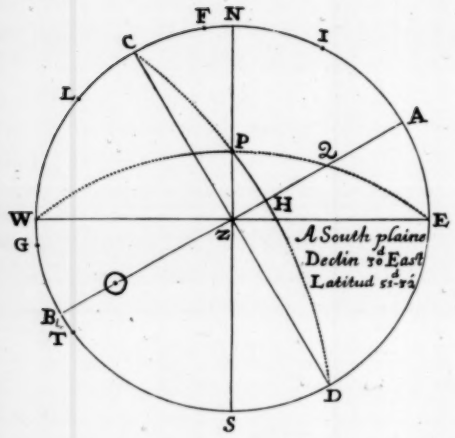
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Declin 70d East
Latitud 51d 32'
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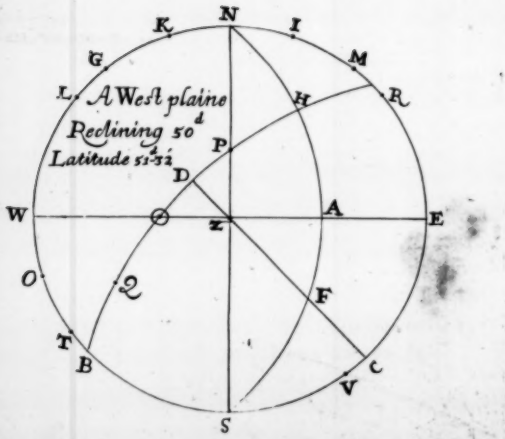
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the Arch WT is the measure of the Inclination of meridians, to wit, $36^d\ 25'$.

4. *Angle of* 12 *and* 6.

In like manner we may draw a Circle passing through the points *W P E*, as the pricke Arch *P E* doth, then in the Triangle *Z P Q* right Angled at *P*, we have the side *ZP* given, and the Angle *PZQ* to finde the side ZQ, lay a Ruler from D to Q, and you will finde a Point in the Limbe, the distance whereof from C is the measure of the Arch sought, to wit, $57^d\ 49'$, to be measured by projection as the Inclination of Meridians.

Lastly the hour-lines, these are represented by meridians drawn through the Poles of the World, as in the Triangle *H P Q* there will be given the Stiles height *PH*, and the Angle *HPQ*, to wit, the Ark of difference between the Inclination of Meridians and the hour from noon, for all hours on that side of the meridian the Substile falls, but on the other side the sum of these two Arks, and this Angle is called the Angle at the Pole; the side required is HQ, the distance between the Substile and the hour line proposed, which must be any hour, though in this Scheam it represents the horary distance of six from the Substile.

To project the Sphere for an East or West Reclining or Inclining Plain, Latitude 51 *degrees*, 32 *minutes, a West Plain, Reclining* 50 *degrees.*

Having drawn the Fundamental Scheam, and therein set off the Latitude as before, count the Reclination from N to R, and by laying a Ruler, finde the point A, through it and the North and South points draw the Arch of a Circle which shall represent the plain, finde the pole thereof by setting off a Quadrant from R to G, then through the pole of that Circle ⊙, and the pole of the World *P*, draw the Arch ⊙*PR*, then in the Triangle *NHP* right angled at H, we have given the side *NP* the Latitude, and the Angle *PNH* the Reclination, to finde *PH* the Stiles height, and NH the Substiles distance from the Meridian, and the Angle NPH the Inclination of Meridians, which is also represented by the Angle *BPS*.

1. *To measure the stiles height.*

Set off a Quadrant from B to C, and draw a Line through the Center, and where it intersects the plain at F, is the Pole of the plains Meridian, lay a Ruler from F to H, and it cuts the Limbe at I.

Also lay it thence to *P*, and it cuts the Limbe at K, the Arch IK is the measure of the Stiles height, to wit, 36d 50′

2. *The Substile.*

A Ruler laid from ⊙ to *H*, cuts the Limbe at M, and the Arch NM is the measure of the Substiles distance from the Meridian, to wit, 38d 59′.

3. *The Inclination of Meridians.*

Set off a Quadrant from K to O, and lay a Ruler from it to F, and it intersects the Meridian of the plain at Q, then a Ruler laid from *P* to Q, findes the point T in the Limbe, and the Arch ST is the measure of the Inclination of Meridians, to wit, 53d 26′.

Otherwise with less trouble lay a Ruler from *P* to F, and it intersects the Limbe at V, and the Arch EV is the measure of the Inclination of Meridians, as before.

To project the Sphere to represent a Declining Reclining Plain.

A South Plain Declining 40 *degrees East, Reclining* 60 *degrees, Latitude* 51 *degrees,* 32 *minutes.*

Having drawn the fundamental Circle, prickt off the Declination, and found the pole point as before, prick the Reclination from B to I, and laying a Ruler to it from A, finde the point R, and through the three points BRA describe a Circle, representing the plain, also from K finde the pole thereof ⊙, and through the two points *P* and ⊙ draw the Arch of a Circle FG, representing the plains Meridian, at the intersection of the plain with the Meridian set *Z*, and draw the Arch of a polar plain through *P* to S, and there will be several Triangles Constituted, from which were derived the several Methods of Calculation.

In the right Angled Triangle ANZ there is given NA the complement of the Declination, and the Angle *NAZ* the complement of the Reclination, whereby may be found *ZN* the plains meridional Ark, which taken from *NP* rests *ZP* the complement of the new Latitude;

tude; alſo the Angle *NZA* which is the complement of the new Declination, and hence were derived the Proportions for the third Method, likewiſe in the ſame Triangle may be found *ZA* the meridians diſtance from the Horizon.

In the Oblique Angled Triangle CPO, there is given the ſide *CP*, the complement of the Latitude, the ſide CO, the complement of the Reclination with the Angle PCO, the complement of the Declination from the South to a Semicircle, whereby may be found the Angle CPO, the Inclination of Meridians, and the Angle COP whereof the meaſure is RH the diſtance, of the Subſtile from the Plains perpendicular, and the third ſide PO, the complement whereof is PH the Stiles height, and from hence was derived the third method of Calculation ſuited to Proportions for finding both the unknown Angles of an Oblique Spherical Triangle at two Operations, when there is given two ſides with the Angle comprehended between them.

The firſt method of Calculation is built upon the perpendicular Trigonometrie, for the perpendicular PS reduceth the former Oblique Triangle, to two right Angled Triangles, to wit, the right Angled Triangle, PSC, and the right Angled Triangle *PSO*, both right Angled at S.

In the right Angled Triangle *PSC*, we have *CP* given the Colatitude, and the Angle PCS the Declination to finde SC a Polar plains Reclination thereto.

Again,

In Oblique Spherical Triangles, reduced to two right Angled Triangles by the demiſſion of a perpendicular, it is a common inference in every book of Trigonometry, when two ſides with the Angle comprehended are given, to finde one of the other Angles: That,

As the Sine of the Side between the Angle ſought and perpendicular,
Is to the Tangent of the given Angle,
So is the Sine of the Side between the Angle given, and perpendicular,
To the Tangent of the Angle ſought.

And ſo in that Oblique Triangle, the difference between the Reclination of the plain propoſed, and the Polar plain is RS, then becauſe RO is a Quadrant, SO is the complement of the former

Ark,

Ark, therefore it holds: As s SO : t SCP :: s SC : t S⊙P which is the very proportion delivered delivered in the said Method for finding the Substiles distance.

Then in the right Angled Triangled PS⊙, we have S⊙, and the Angle S⊙P to finde the side P⊙, whereby is got the Stiles height; the Inclination of Meridians is found in the Oblique Spherical Triangle by the proportion of Opposite sides and Angles.

Lastly, in the right Angled Triangle ZRC, there is given RC, and RCZ to finde RZ the distance of the Meridian from the plains perpendicular.

To measure the respective Arks abovesaid.

1. *The New Latitude.*

A Ruler laid from W to Z, and P will give you the Arch MO in the Limbe, the complement of the new Latitude, to wit, 27^d 40′.

2. *The New Declination.*

The Ruler laid from the Plains Zenith at Z, to its Pole at ⊙, findes the point Q in the Limbe, and the Arch SQ is the new Declination, to wit, 18^d 56′.

3. *The Substiles distance from the Plains perpendicular.*

A Ruler laid from ⊙ to *H*, findes the point T in the Limbe, and the Arch CT being 26^d 26′ is the Substiles distance from the Plains perpendicular.

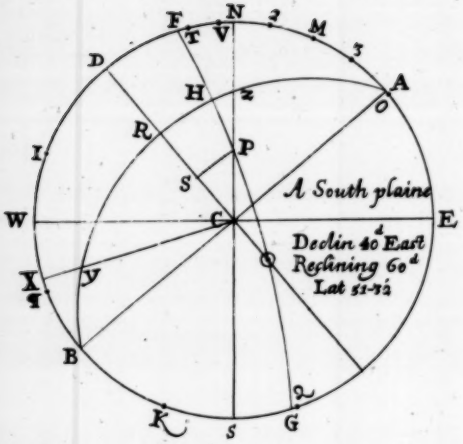
4. *The Meridians distance from the Plains perpendicular.*

A Ruler laid from ⊙ to Z, findes the point V in the Limbe, and the Arch DV being 35^d 56′ is the Meridians distance from the Plains perpendicular.

5. *The Stiles height.*

Set off a Quadrant from G to X, and draw XG, where it intersects the plain as at Y, is the Pole of the Arch FG, then laying a Ruler from Y to H and P, you shall finde the Stiles height in the Limbe to be the Arch 2, 3, namely, 26^d 6′.

6. *In-*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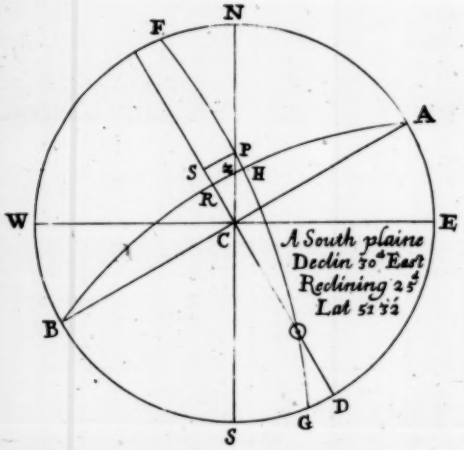
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6. The Inclination of Meridians.

A Ruler laid from *P* to ♈, interſects the Limbe at ¶, the Arch W¶ is 20ᵈ 58′, and ſo much is the Inclination of Meridians.

The Polar Reclination CS is 31ᵈ 19′.

The Scheam determineth all the affections of the plain.

1. It ſhews that H the point of the Subſtilar lies on that ſide the plains perpendicular, that is towards the Coaſt of Declination.

2. That Z the point for the place of the Meridian, lyes towards the ſame Coaſt as before, but below the Subſtilar Line.

3. The Arch PH ſhews you that the North Pole is elevated above the upper or Reclining face.

After the ſame manner may all the Requiſite Arks be meaſured, and affections determined for all plains whatſoever.

A South Plain Declining 30 *degrees Eaſtwards, Reclining* 25 *degrees, or a North Plain Declining* 30 *degrees Weſtwards, Inclining* 25 *degrees.*

In this Scheam we have the ſame Oblique Triangle PC⊙ reduced to two right Angled Triangles *PS*♑ and *PS*⊙, S♑ is the Inclination of a Polar plain, and RC the Inclination of the plain propoſed, the difference is SR, and the complement of it, is the complement of S⊙ to a Semicircle, becauſe S⊙ is greater then a Quadrant, and the proportions are wholly the ſame, though the Triangle have ſides greater then a Quadrant.

The North Pole is elevated on the Inclining face, the Meridian Z lyes from the plains perpendicular towards that end of the Horizontal Line, oppoſite to the Coaſt of Declination, the ſame way and beneath it lyeth the Subſtilar.

The complement of the new Latitude *ZP* is 10ᵈ 10′
The new Declination, *viz.* the complement of NZA is 26ᵈ 57′
The meridians diſtance from the Plains perpendicular RZ 13ᵈ 43′.
The Subſtiles diſtance therefrom RH 18ᵈ 22′
The Stiles height PH is ——— 5ᵈ — 3′
The Inclination of Meridians, to wit, the Angle *CP*⊙ 27ᵈ 18′
The Polar Reclination ——— CS — 34ᵈ 31′

A

A South Plain Declining 40 *degrees East, Inclining* 15 *degrees, or rather a North Plain Declining* 40 *degrees West, Reclining* 15 *degrees.*

Here again the Oblique Triangle *CP*⊙ is reduced to two right Angled Triangles *PSC* and *PS*⊙, and SR is the sum of the Polar Reclination SC, and the $\left.\begin{matrix}\text{Re}\\ \text{In}\end{matrix}\right\}$clination of the Plain proposed CR, and S⊙ is the complement hereof, because ⊙R is a Quadrant, finde the Equinoctial point *Æ*.

The Polar Reclination CS 31^d 19$'$.

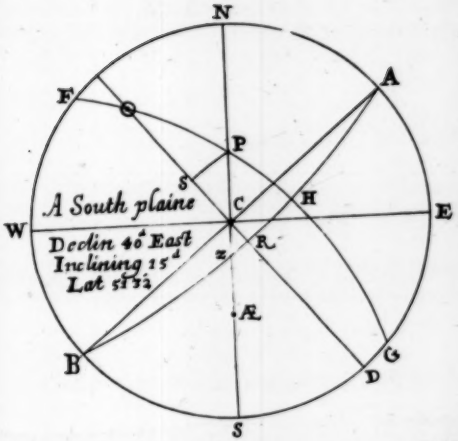
The new Latitude Z*Æ* is 32^d 15$'$.
The new Declination being the complement of *NZA* is 38^d 23$'$
The Meridians distance from the plains perpendicular ZR 12^d 33$'$.
The Substiles distance from the plains perpendicular RH 32^d 16$'$
The Stiles height PH is ——————————— 41^d 30$'$
The Inclination of Meridians ⊙PC rather the Acute Angle ⊙P*N* is } 55^d 58$'$

The Meridian Z lies from the plains perpendicular towards the Coast of Declination on the Reclining side, but must be drawn through the Center, because the Sun at noon casts his shadow Northwards, unless in the Torrid or Frozen Zone, and the Substile H lyes on the other side the plains perpendicular.

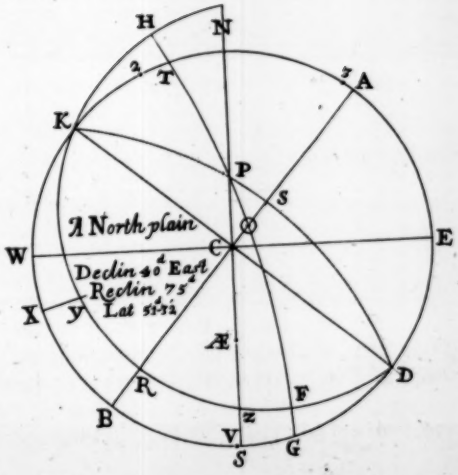
A North Plain Declining 40 *degrees Eastwards, Reclining* 75 *degrees.*

In this plain likewise the Oblique Triangle *C*⊙*P* is reduced to two right Angled Triangles *PS*⊙ and *PSC* by the perpendicular *PS* which is part of the Arch of a polar plain, here CR more CS is equal to the sum of the Plains Reclination proposed, and of the Polar plains Reclination, which is greater then a Quadrant for the Arch R⊙ is a Quadrant; now the Cosine of an Ark greater then a Quadrant is the Sine of that Arks excess above a Quadrant, wherefore the Sine of S⊙ is the Cosine of the sum of both the Reclinations, and the Case the same as before.

The Arch RH is the measure of the Substiles distance from the plains


N
A
F
P
S
A South plaine
C
H
E
W
Declin 40d East
Inclining 15d
Lat 51d 32'
z
R
Æ
B
G
D
S

page 56


H
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A North plain
E
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C
Declin 40d East
Reclin 75d
X
y
Lat 51d 32'
Æ
D
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S

Plains perpendicular, which being greater then a Quadrant, ſhews that the Subſtile muſt lye below the end of the Horizontal Line oppoſite to the Coaſt of Declination, to wit, it muſt be depreſſed $32^d\ 39'$ meaſured by the Arch KT, the complement of TA its diſtance from the plains perpendicular, the point T being found by laying a Ruler from ⊙ to H, which may alſo be meaſured from F, and will fall above the Horizontal Line D, but then muſt be drawn through the Center.

Prick off a Quadrant from G to X, and draw XC, it cuts the plain at Y, a Ruler laid from Y to H, and *P* findes the points 2, 3 in the Limbe, and the Arch 2, 3 being $61^d\ 31'$, is the Stiles height; orthet complement thereof to a Semicircle might be found by meaſuring the Arch PF.

A Ruler laid from ⊙ to *Z*, findes the point V in the Limbe for the Meridian Line, from which draw a Line through the Center on the other ſide, and it will be placed in its true Coaſt and quantity from the Plains perpendicular at A, to wit, $39^d\ 2'$.

The Inclination of Meridians, to wit, the Angle $CP⊙$ is $20^d\ 30'$,

The new Latitude $ZÆ\ 26^d\ 38'$.

The new Declination is $9^d\ 35'$ to wit, the complement of KRB.

The Polar Reclination CS is $31^d\ 19'$.

The truth of this Stereographick Projection is fully handled by *Aguilonius* in his Opticks, and how to determine the affecton of any Angle of an Oblique Spherical Triangle, I have fully ſhewed in a Treatiſe, called the *Sector on a Quadrant*.

For the Reſolution of Spherical propoſitions, Delineations from proportions or the Analemma, will be more ſpeedy and certain (though they may alſo be thus reſolved) which I have handled at large in the *Mariners Plain Scale new plain'd.*

To determine what hours are proper to all kinde of Plains.

To do this it will be neceſſary to project upon the Plain of the Horizon, the Summer and Winter Tropicks.

Get the Sum and difference of the Colatitude, and of the Suns greateſt Declination, ſo we ſhall obtain his greateſt and leaſt Meridian Altitudes.

The depreſſion of the Tropick of *Cancer* under the Horizon, is

equal to the leaſt Meridian Altitude, and the depreſſion of the Tropick of *Capricorn* to the greateſt. Example:

38d 28′ Colatitude
23d 31′

61d 59′ greateſt } Meridian Altitude, having drawn the Primitive
14d 57′ leaſt } Circle, &c. as before.

Prick 14d 15′ from *S* to C, and 61d 59′ from *S* towards W, a Ruler laid from the points found, will interſect the meridian *ZS* at the point L for the Winter Tropick, and K for the Summer Tropick, through which the Circles that repreſent them are to paſs, to finde the Semidiameters whereof, ſet off their depreſſion from N towards E, thus 14d 57′ the depreſſion of the Summer Tropick terminates at O, a Ruler laid from E to ⊙, findes the point X in the meridian SZN produced, ſo is XK the Diameter of the Summer Tropick, which being divided into halfs, will finde the Center thereof whereon to deſcribe it.

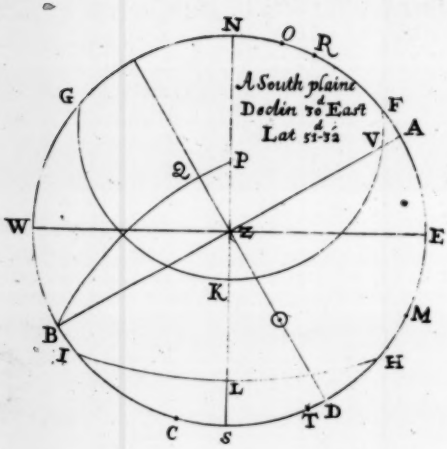
In like manner is the Diameter of the Winter Tropick to be found; or if the Amplitude be given (or found as elſewhere is ſhewed) which at *London* is 39d 54′, and ſet off both ways from G and E we ſhall have three points given through which to draw each Tropick, and the Centers falling in the Meridian Line will be found with half the trouble, as to finde a Center to three Points.

Alſo Project the Pole Point *P* as before, being thus prepared FKG will repreſent the Summer and HLI the Winter Tropick.

Let it be required to know what hours are proper for a South plain Declining 30d Eaſtwards, through the three points BPA deſcribe the the Arch of a Circle *BQP*, then laying a Ruler from B to *Q*, finde the point R in the Limbe, and from it ſet off a Quadrant to M, then a Ruler laid from B to M findes the point ⊙ the Pole of the hour Circle *BQP*, then laying a Ruler from *P* to ⊙, it findes the point T in the Limbe, and the Arch ET being 65d 4′ is the meaſure of the Angle BPS, which turned into Time is 4 ho. 23′ prope, and ſheweth that at no time of the year the Sun will ſhine longer on the South ſide of this plain, then 23 minutes paſt 4 in the afternoon.

In like manner if the Arch of a Circle be drawn through the two points *PV*, we may finde the time when the Sun will ſooneſt in the morning begin to ſhine on the South ſide of this plain.

The Angle *QPZ* may be found by Calculation in the right Angled


N
O
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A South plaine
Declin 30d East
Lat 51-32
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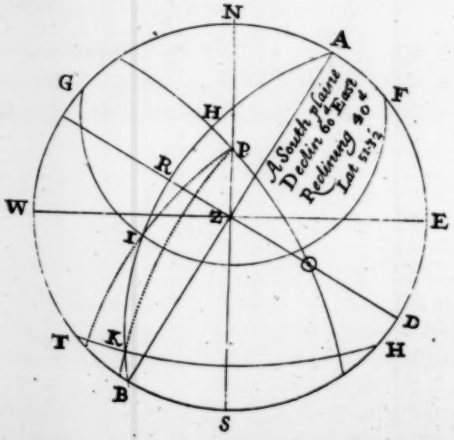
N
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A South plaine
Declin 60 East
Reclining 40 d
Lat 51-32
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K
H
B
S

gled triangle *PQZ*, in which beſides the right Angle at *Q*, the ſide *PZ* and the Angle *QZP*, are given but to meaſure it off by Projection will be ſufficient for the Dyalliſts uſe.

So if there were a South plain Declining 60^{d} Eaſtwards, Reclining 40 degrees here repreſented by BRA, if it were required to know what hours are proper for the upper, and what for the under face, then where the plain interſects the Tropicks as at I and K, draw two Meridians into the Pole at *P*; to wit, IP and *KP*, and firſt finde the Angle *IPZ*, as was before ſhewed, to wit, 53^{d} 14′ which in time is 3 hours 33 minutes, ſhewing that the Sun never ſhines longer on the upper face of the Plain, then 33 minutes paſt 3 in the afternoon, which is capable of receiving all hours from Sun riſing to that period of time, and the Angle KPZ, to wit, 39^{d} 50′ in time 2 hours 39 minutes, ſhews that the Sun never begins to ſhine ſooner on the under face then 39 minutes paſt 2 in the afternoon after, which all the hours to Sun-ſet may be expreſſed.

To finde theſe Arks of time by Calculation, there muſt be given the Stiles height above the Plain 15^{d} 22′, *PH*, and the complement of the Inclination of Meridians to a Semicircle, 136^{d} 32′, to wit, *HPS*, then in the right Angled Triangle *PHI* there is given *PH* the Stiles height, PI the complement of the Declination, beſides the right Angle at H, to finde the Angle *IPH* 83^{d} 18′ which taken from the complement of the Inclination of Meridians *HPS*, there reſts the Angle *IPS* the Arch of time ſought, to wit, 53^{d} 14′..

The aſcenſional difference may be found by drawing the Arch of a Circle through the three points *TPF*, and thereby the length of the longeſt day determined that no hours be expreſſed, on which the Sun can never ſhine.

Another manner of Inſcribing the hour-lines in all Plains having Centers.

The method here intended, is to do it in a Parallelogram from the Meridian line, whence the hour-lines may be prickt down by a Tangent of three hours, with their halfs and quarters from a Sector, without collecting Angles at the Pole, or by help of a Scheam which I call the Tangent Scheam, the foundation of this Dyalling ſuppoſeth the Axis of the world to be inſcribed in a Parallelipipedon continued about the Axis, the ſides whereof are by the plains of the reſpective

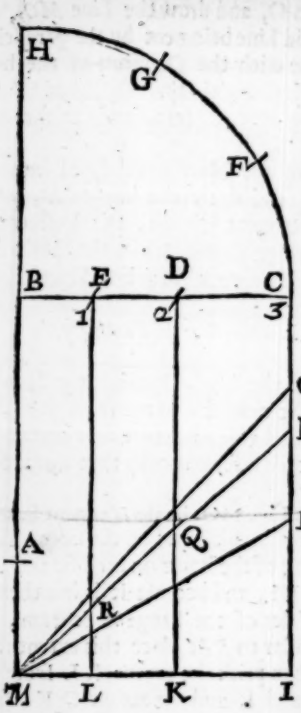
ſpective hour Circles in the Sphere divided into Tangent-lines, that is to ſay, each ſide is divided into a double tangent of 45ᵈ ſet together in the middle, and the ſaid parallelipipedon being cut by any Plain, the end thereof ſuppoſed to be interſected, ſhalbe either a right or Oblique Angled parallelogram, and then if from the oppoſite tangent hourpoints on the ſides of the interſected parallelipipedon, lines be drawn on the Plain, they ſhall croſs one another in a Center, and be the hour-lines proper to the ſaid plain, but of the Demonſtration hereof, I ſhall ſay no more at preſent, the inquiſitive Reader will finde it in the Works of *Clavius*.

To draw the Tangent Scheam.

I have before in Page 10 ſhewed how to divide a Tangent Line into hours and quarters, which in part muſt be here repeated; draw any right Line, as *MABH*, from any point therein as at B, raiſe a perpendicular, and upon B as a Center, deſcribe the Quadrant CH, and prick the Radius from B to A, from C to G, from H to F, and laying a Ruler from A to F, and G, you will finde the points D and E upon the perpendicular CB, I ſay the ſaid perpendicular is divided into a Tangent line of three hours, and the halfs and quarters may be alſo divided thereon, by dividing the Arches CF, FG, and GH into halfs and quarters, then from thoſe ſubdiviſions, laying a Ruler to A, the halfs and quarters may be divided on, as were the whole hours.

Being thus prepared, draw the Lines *MB*, *LE*, *KD*, and *IC*, all parallel one to another, paſſing through the points B, 1, 2, 3.

In



In this Scheam they are perpendicular to BC, but that is not material, provided they pass through the same points, and are parallel one to another, yet notwithstanding the points A and H must be in a right Line perpendicular to CB. This Scheam thus prepared, I call the Tangent Scheam, because a Line ruled any way over it, shall be divided also into a Tangent of the like hours and quarters, whence it follows that one of these Scheams may serve to inscribe the hour-lines into many Dyals, which I shall next handle.

To inscribe the Hour-lines in a Horizontal Dyal.

Having drawn the Meridian line M, XII, and perpendicular thereto the hour-line of six, Let it be observed that the sides of the Horizontal Dyal in page 8. to wit VI, IX, and VI, III and IX, III, are a right Angled Parallelogram, the one side whereof being the Diameter of the Circle being Radius, the other side thereof must be made equal to the Sine of the Latitude, in that Scheam the neareſt distance from L to MF was the Sine of the Latitude or Stiles height, the Semidiameter of the inward Circle, being Radius, and that extent being doubled and pricked from M to VI on each side, as also from F twice to IX and III, by those extents the Parallelogram was bounded, the side III, F IX, being parallel to the Horizontal Line.

Then take the extent MF from the Horizontal Dyal, and place it in

in the Tangent Scheam from M to O, and draw the Line *MO*, and the reſpective Diviſions of the ſaid Line being cut by the parallels of the Tangent Scheam are the ſame with the Diviſions of the hour-lines on the inward ſides of the Horizontal Dyal VI,IX,and VI,III, and from the tangent Scheam they are to be transferred thither with Compaſſes.

Alſo place F, IX or F, III from the Horizontal Dyal into the Tangent Scheam from *M* to *N*, and draw the line *MN*, which being cut by the parallels of the tangent Scheam, the Diſtances of thoſe Diviſions from *M* are to be pricked down in the Horizontal Dyal,the firſt from F to XI,and I the ſecond from F to X and II,&c.

To inſcribe the hour-lines in a direct erect South Dyal.

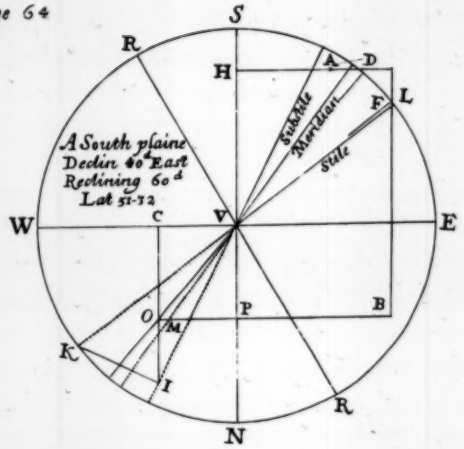
The Diameter of the Circle in the South Dyal in page 8, is the ſame as in the Horizontal, and the neareſt diſtance from L to *FM* was the Coſine of the Latitude, and was pricked twice on the Horizontal Line from *M* to VI on each ſide, whereby that inward Parallelogram was limited.

Wherefore the diviſions of the Line MO in the Tangent Scheam, are the ſame with the hour diſtances in this Dyal on the ſides VI, III, and VI, IX; then for the diviſions of the hours on each ſide of XII, take the extent XII, IX or XII, III, and becauſe it is leſs then the outward parallel diſtance of the ſides of the Tangent Scheam, having therein made *MI* perpendicular to *BM*,place this extent from I to P, and draw the line *MP*, then prick the extent RL from the Tangent Scheam, from F to XI and I, and the extent QK, from F in the Dyal, to X, and II, and from the hour points ſo found, draw lines into the Center at *M*, and they ſhall be the hour-lines required.

To delineate an upright Decliner in an Oblique Parallelogram.

An upright South Dyal Declining 30 degrees Weſt, Latitude 51 degrees, 32 minutes.

Firſt draw the Meridian or Plains perpendicular CN, and upon C as a Center, with the Radius of the Dyalling Scheam, deſcribe an



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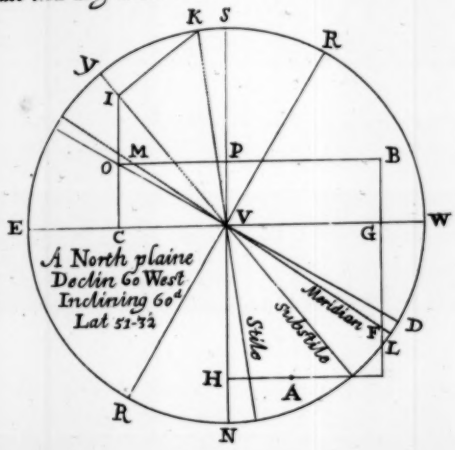

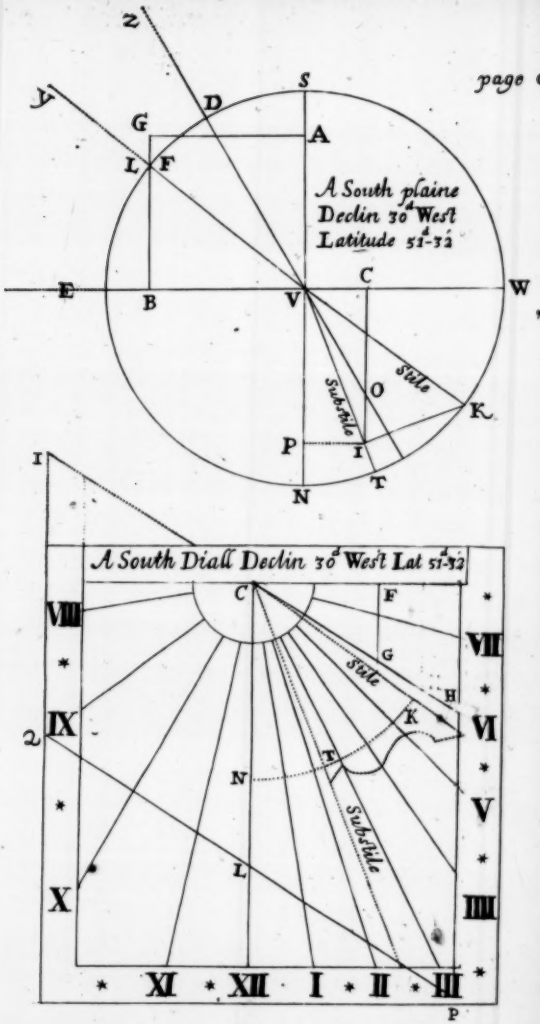
Z
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L
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A South plaine
Declin 30d West
Latitude 51d-32'
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A South Diall Declin 30d West Lat 51d-32'
VIII
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an Occult Ark, wherein prick off NT from the aforeſaid Scheam, and draw TC for the Subſtile; alſo ſet off in the ſaid Ark TK equal to TK of the beforementioned Scheam, and draw KC for the Stile; perpendicular to the Meridian paſſing through the Center C, draw the Plains Horizontal Line, and therein from the Center prick down from C to F, the Cotangent of the Latitude, to wit, BF upon which the Sine of the Declination is to be erected to the ſame Radius, and the neareſt diſtance from A to DV in that Scheam, is the Sine of the declination, which being prickt on the perpendicular FG, draw a Line from the point G into the Center, and it ſhall be the hour-line of ſix.

In Latitudes under 45^d the ſide of the ſquare AV muſt be aſſumed to be the Cotangent of the Latitude, the Radius whereto will be AF the Tangent of the Latitude, to which Radius being prickt on the ſide of the ſquare from the Center, the Sine of the declination muſt be taken out as before, and erected on the Cotangent of the Latitude, and this work muſt be performed on that ſide of the Center on which the Subſtile lyes.

To fit in the Parallelogram.

Produce the Lines WE, VD, and VL, in the Dyalling Scheam far enough, then aſſuming any extent to be Radius, enter it on the lines VD and VL from the Center to Y and Z, the neareſt diſtance from Y to VE is the Coſine of the Latitude to that Radius which enter on the Line of 6 or GC, ſo that one foot reſting thereon, the other turned about may juſt touch CN, at the point found ſet H, and make CI on the other ſide equal to CH.

The neareſt diſtance in the Dyalling Scheam from Z to VE, is the Coſine of the declination to the former Radius, which prick on the Meridian line from C to L.

And draw a Line through L parallel to HI, and therein make LP, LQ each equal to C H, and draw HP and IQ, and there will be an oblique Parallelogram conſtituted, the ſides whereof will be Tangent Lines.

Nota, we might aſſume the point G in the hour-line of ſix, for the Parallelogram to paſs through, and the neareſt diſtance from D to VE in the dyalling Scheam, would be the Coſine of the declinati-

on

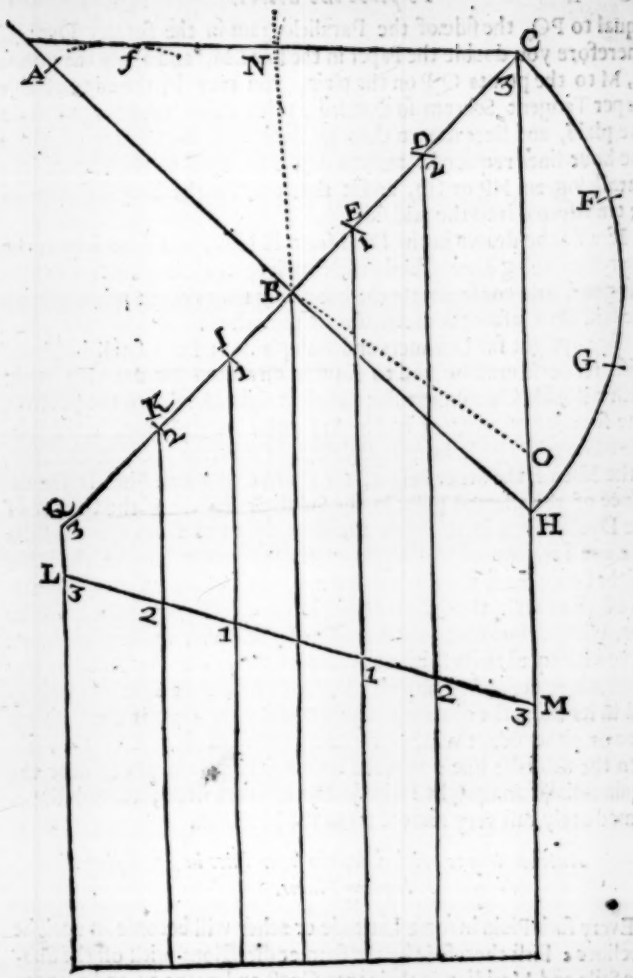
on to the ſame Radius to be prickt on the Meridian Line as before.

To inſcribe the hour-lines.

In the following Tangent Scheam made as the former, produce CB, and make BQ equal to BC, then take the extent PQ on the dyal, and upon Q as a Center deſcribe the Ark Y therewith, and draw the Line YC juſt touching the extreamity, then you may proportion out the hours in this manner.

The neareſt diſtance from *D* to YC in the Tangent Scheam, being prickt on the ſide of the Parallelogram from P and Q, will finde the points 10 and 2, through which thoſe hour-lines are to paſs, alſo the neareſt diſtance from E to YC, being there prickt, findes the points 11 and 1 as before, otherwiſe take the extent PL from the Dyal, and in the Tangent Scheam upon C as a Center, deſcribe the Ark N therewith, and draw BN juſt touching the outward extreamity of that Ark, the neareſt diſtance from E to BN, prick in the Dyal from L to 11 and 1, and the neareſt diſtance from D to BN, place from L to 10 and 2, thus you have found points through which part of the hour-lines are to be drawn into the Center at C; or having found the points P and Q on the plain, if it be deſired to inſcribe the hours without drawing the ſide of the Parallelogram PQ, which muſt alſo be performed without the help of Compaſſes, though they be at hand, then in the Tangent Scheam prick the extents E and D, from B to I and K, and through the reſpective points C, D, E, B, I, K, Q, draw lines all parallel one to another, making any Angle at pleaſure with the ſaid Line, I ſay then that any line ruled over thoſe parallels, ſhall be divided into a line of double tangents, as is LM, which is made

equal



K equal

equal to PQ, the ſide of the Parallelogram in the former Dyal, if therefore you double the Paper in the line LM, and apply the points L, M to the points Q P on the plain, you may by the edge of this Paper Tangent Scheam ſo doubled, prick down the hour points on the plain, and lines drawn through them into the Center, ſhall be the hour-lines required, and making B O equal to the ſides of the Parallelogram HP or IQ, and it ſhall contain the hour diſtances to be transferred into the ſaid ſides.

If a line be drawn in the Dyal from H to L, and from L to I, the hour-lines being drawn ſhall divide each of theſe lines into a double Tangent, and conſequently the hour-lines may alſo be prickt off on the ſaid lines, after the method now preſcribed.

For upright far Decliners and ſuch plains as have ſmall height of Stile, recourſe muſt be had to former directions for drawing them with a double Contingent line, each at right Angles to the Subſtile. The foundation whereof is this, Any point being aſſumed in the Subſtilar line of a Dyal, the neareſt diſtance from that Point to the Stile, is the Sine of the Stiles height, the Radius to which Sine is the diſtance of the aſſumed point in the Subſtilar line from the Center of the Dyal; Then in all Dyals the hour diſtances from the Subſtilar line are Tangents of the Angle of the Pole, the Sine of the Stiles height being made the Radius thereto, having finiſhed the delineation of the Dyal, the Stile is to be placed directly over the Subſtilar line, without inclining to either ſide of the Plain, making an Angle therewith equal to its height above the ſame, for the Subſtilar line is elſewhere defined to be ſuch a line over which the Stile is to be placed in its neareſt diſtance from the Plain, therefore if the Stile incline on either ſide, it will be nearer to ſome other part of the Plain then the Subſtilar line, whence it comes to paſs in places near the Equinoctial, if an upright Plain decline but very little, the Subſtile is immediately caſt very remote from the Meridian.

Another Way to prick down the hour-lines in Declining leaning Plains.

Every ſuch Plain in ſome Latitude or other will become an upright Decliner: Firſt therefore by the former directions prick off the Subſtile, Stile and Meridian, in their true Coaſt and quantity, and perpendicular to the Meridian, draw a line paſſing through the Center, and it

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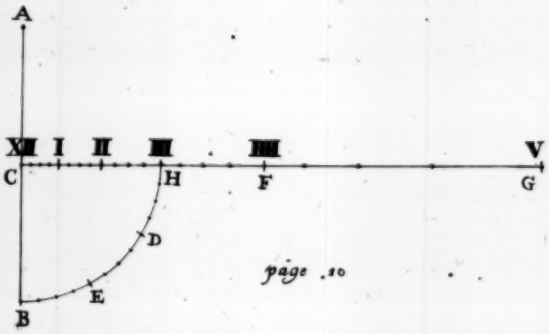

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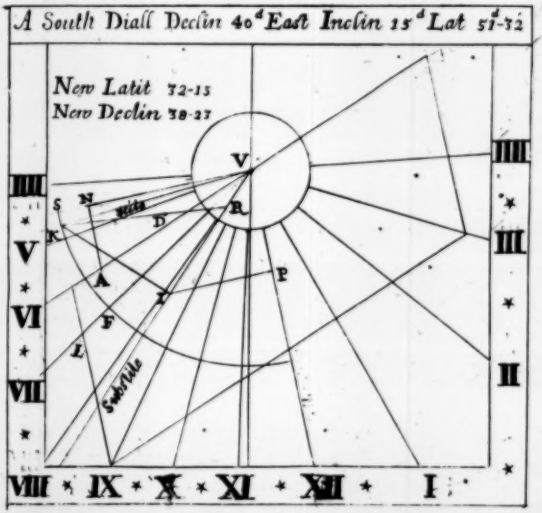

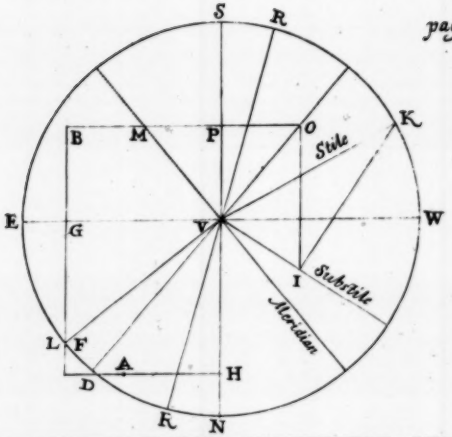
S
R
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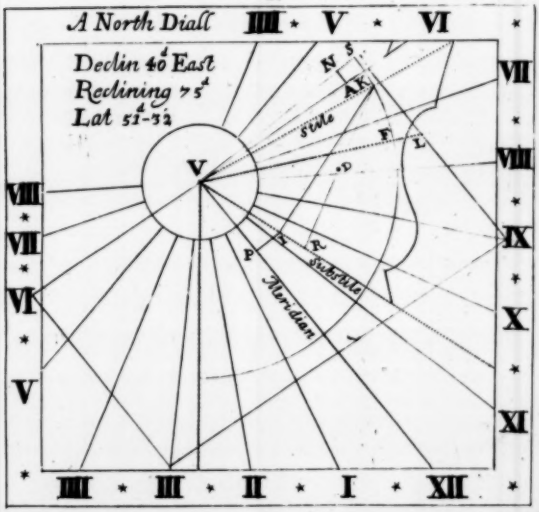

A North Diall
Declin 40d East
Reclining 75d
Lat 51d-32
IIII
V
VI
VII
VIII
IX
X
XI
XII
I
II
III
IIII
V
VI
VII
VIII
N
S
A
K
Stile
F
L
D
V
P
R
Substile
Meridian

it shall represent the Horizontal line of the Plain in that new Latitude. as here VS; from any point in the Stile as K, let fall a perpendicular to the Substile at I, and from the point I, in the Substile let fall a perpendicular to the Meridian at P.

To finde the new Declination.

Prick IP on the substilar line from I to R, and draw RK, so shall the Angle IRK be the complement of the new declination, and the Angle IKR the new Declination it self.

To finde the new Latitude.

Upon the Center V with the Radius VK, describe a Circle, I say then that VP is the Sine of the new Latitude to that Radius which may be measured in the Limbe of the said Circle, by a line drawn parallel to VS, which will intersect the Circle at F, so is the Arch SF the measure of the new Latitude.

To prick off the Hour-line of Six.

This must be prickt off below the Horizontal line, the same way that the substilar lyes: The proportion, is,

As the Cotangent of the Latitude, Is to the Sine of the Declination, So is the Radius, To the Tangent of the Angle between the Horizon and six.

If RK be Radius, then is VP the Tangent of the new Latitude, but if we make VP Radius, then is RK the Cotangent of the new Latitude.

Wherefore prick the extent RK on the Horizontal line from V to N, and thereon erect the Sine of the Declination to the same Radius perpendicularly, as is NA, and a line drawn into the Center shall be the hour-line of six; the proportioning out of the Sine of the Declination to the same Radius, will be easily done, enter the Radius VP from K to D, and the neareſt distance from D to IK, shall be the Sine of the new declination to that Radius.

To fit in the Parallelogram.

This is to be done as in upright Decliners, for having drawn a line

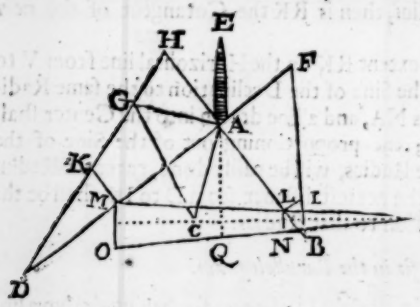
from the new Latitude at F into the Center, if any Radius be entred on the said line from V, the Center towards the Limbe, the neareſt diſtance from that point to VP the Meridian line, ſhall be the Coſine of the new Latitude to that Radius.

Again if the ſame Radius be entred on RK produced if need be, the neareſt diſtance to VR (produced when need requires) ſhall be the Coſine of the new Declination, and then the hour-lines are to be drawn as for upright decliners, nothing will be doubted concerning the truth of what is here delivered, if the demonſtration for inſcribing the Requiſites in upright decliners be well underſtood, it being granted that Oblique Plains in ſome Latitude or other will become upright decliners.

There are two Examples for the Latitude of *London* ſuited to theſe directions, in both which the Letters are alike, the one for a South Plain declining 40^{d} Eaſtwards, Inclining 15^{d}, the other for a North Plain declining 40^{d} Eaſt, Reclining 75^{d}.

To finde a true Meridian Line.

For the true placing of an Horizontal Dyal, as alſo for other good uſes it will be requiſite to draw a true Meridian Line, which propoſition may be performed ſeveral ways, amongſt others the Learned Mathematician *Francis van Schooten* in his late *Miſcellanies* demonſtrates one, performed by help of three ſhadows of an upright Stile on a Horizontal Plain, publiſhed firſt without Demonſtration in an *Italian* book of dyalling, by *Mutio Oddi*.



Let AB, AC, and *AD* repreſent 3 ſhadows made on a Horizontal plain in one day, by the ſhadow of the ſtile or wyre AE, erected perpendicularly from the point A, of which ſhadows if two be found equal, a line let fall perpendicularly from

from the Point A, upon a line joyning the extreamities of thoſe two ſhadows, ſhall be a true Meridian line.

But if all three be unequal, as let AC be the leaſt, erect three lines from the point A, perpendicular to AP, AC, AD as is AF, AG, and AH equal to the Stiles height AE, and draw lines from the extreamities of the three ſhadows to theſe three points as are FB, GC, and HD; then becauſe AC is leſs then AB, therefore GC will be leſs then FB, by the like reaſon GC will be leſs then HD, wherefore from FB and HD cut off or Subſtract FI and HK equal to GC, and from the points I and K let fall the perpendiculars IL, KM, upon the Baſes AB, AD, afterwards draw a line joyning the two points M, L, and from the ſaid points let fall the perpendiculars LN equal to LI, and MO equal to MK.

Then becauſe the two ſhadows AB and AD are unequal, in like manner FB and HD will be unequal; but foraſmuch as FI and HK, are equal by conſtruction, it follows that LI, KM, or LN and MO will be unequal, and foraſmuch as theſe latter lines are parallel a right line that connects the points O and N, being produced will meet with the right line that joyns M, L produced, as let them meet in the point P, from whence draw a Line to C, and it ſhall be a true line of Eaſt and Weſt, and any Line perpendicular thereto ſhall be a Meridian line, thus the perpendicular AQ let fall thereon, is a true Meridian Line paſſing through the point A, the place of the ſtile or wyre.

Whereto I adde that if MO & LN retaining their due quantities be made parallel it matters not whether they are Perpendicular to ML or no, alſo for the more exact finding the Point P, the lines MO and LN, or any other line drawn parallel to them, may be multipyled or increaſed both of them the like number of times from the points *M* and *L* upwards, as alſo from the points O and N downwards, and Lines drawn through the points thus diſcovered, ſhal meet at P without producing either *ML* or ON. See 15 Prop. of 5 Euclid. and the fourth of the ſixth Book.

The greater part of *van Schootens* Demonſtration is ſpent in proving that *ML* and ON produced will meet ſomewhere, this for the reaſons delivered in the conſtruction I ſhall aſſume as granted, then underſtand that the three Triangles ABF, ACG and ADH ſtand perpendicularly erect on the plain of the Horizon beneath them, upon

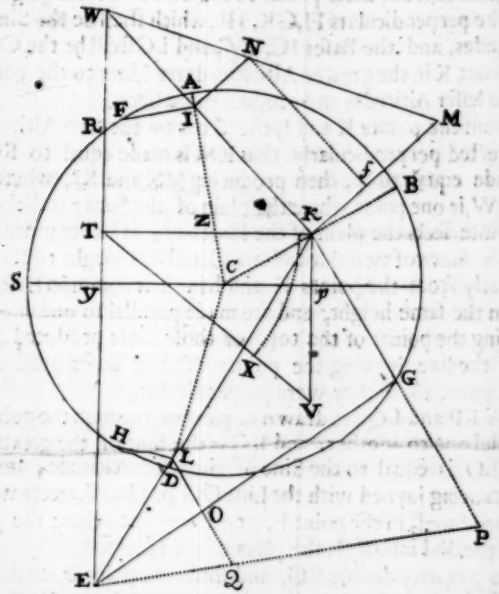
on the right Lines AB, AC and AD, whence it will come to pass that the three points F, G, and H meet in one point, as in E the top of the Stile AE, and that the right lines FB, GC and HD are in the Conique Surface of the shadow which the Sun describes the same day by his motion, the top of which Cone being the point E.

Wherefore if from those right Lines we substract or cut off the right Lines FI, GC and *HK* being each of them equal to one another, then will the points I, C and K, fall in the circumference of a Circle, the plain whereof is parallel to the plain of the Equator; and therefore if through the points K and I in such a Position a right Line be imagined to pass, and be produced to the plain of the Horizon, it will meet with *ML* produced in the point *P*, where ON being produced, will also meet with it; so that the point P being in the Plain of that Circle, as also in the plain beneath it, as also the point C being in each Plain, a right line drawn through the Points P and C will be the Common Intersection of each plain, and the line PC will be parallel to the Plain of the Equator, and is therefore a true Line of East and West, which was to be proved.

On all Plains though they Decline, and Recline, or Incline, after the same manner may be found the Line PC, which will represent the Contingent Line of any Dyal, and a perpendicular raised upon the Line CP shall be the Substilar Line, which in Oblique Plains is the Meridian of the Plain, but not of the place, unless they are both Coincident from this manner of finding a Meridian Line on a Horizontal plain, nothing else can be deduced without more Scheams: From the three shadows, may be had the three Altitudes, and the Meridian line being given, the Azimuths to those three shadows are likewise given, which is more then need be required in order to the finding of the Latitude of the place, and the Declination and Amplitude of the Sun, which because this Scheam doth not perform of it self, I shall adde another to that purpose.

By three Altitudes of the Sun, and three shadows of an Index on an Horizontal Plain, to finde a true Meridian Line, and consequently the Azimuths of those shadows, the Latitude of the place, the Suns Amplitude and Declination.

It



It will be convenient to take two shadows, when the Sun is on one side of the Meridian, and the third when on the other side, and the performance will be most exact when the Sun is near the Summer Solstice.

Let the three shadows be CA, the Altitude whereto is AF 22^d, 28$'$
The second shadow CB, the Altitude whereto is —— BG 59^d, 21$'$
both these in the morning,
The third shadow in the afternoon CD the Altitude whereto is —— DH 18^d, 20$'$

Let the Angle ACB be 70^d, and the Angle BCD 135^d, by following Operations we shall finde that the shadow CA, is 10^d to Southwards of the West, that the shadow CB is 60^d to Northwards of the West, that the shadow CD is 15^d to Southwards of the East.

Having

Having from the three ſhadows prickt off the three Altitudes to F,G,and H,from thoſe points to the ſhadows belonging to them, let fall the perpendiculars FI,GK, HL,which ſhall be the Sines of thoſe Altitudes, and the Baſes IC, *KC*, and LC ſhall be the Coſines,from the point K in the greater Altitude, draw Lines to the points I and L in the leſſer Altitudes and produce thoſe lines.

From the points K and I, the Sines of the two Altitudes, are to be erected perpendicularly, thus KM is made equal to KG, and IN is made equal to IF, then produeing MN and KI, where they meet as at W,is one point,where the plain of the Suns parallel of Declination interſects the plain of the Horizon; in like manner on the Baſe KL,the Sines of two Altitudes KG,and LH, ought to ſtand perpendicularly from the points K and L in their common Baſe, but if they retain the ſame height, and are made parallel to one another, a line joyning the points of the tops of thoſe Sines produced, ſhall meet with the line joyning the points of their Baſes produced, in the ſame point as if they were perpendicular.

Thus KP and LQ are drawn at pleaſure through the points K,and L parallel one to another, and KG is the Sine of the greater Altitude, and LO is equal to the Sine of the leſſer Altitude, and theſe two points being joyned with the Line OG produced,meets with the Line KL produced, in the point E, another point where the plain of the Suns parallel interſects the plain of the Horizon.

Or you may double KG, and finde,the point *P*,as alſo double LO and finde the point Q, a line drawn through *P* and *Q*, findes the point E, as before, but with more certainty.

If you joyn EW it ſhall be a true Line of Eaſt and Weſt, and a perpendicular let fall thereon from C the Center, ſhall be a true Meridian line to the perpendicular Stile or wyer,as is C S.

The Arch SR is the Suns Amplitude from the North 50^{d} 6′ the ſhadow being contrary to the Sun, caſts his parallel towards the South.

Draw KT parallel to SC, and it ſhall be the perpendicular diſtance between the Sine of the Suns greateſt Altitude, and the Interſection of his parallel with the Horizon.

Upon *K* erect *KV* equal to the Sine of the Suns greateſt Altitude of the three *KG*, and the Angle *KTV* ſhall be equal to the complement of the Latitude, for the Angle between the plain of any parallel

rallel of declination and the Plain of the Horizon, is always equal to the complement of the Latitude, and if upon T as a Center with the Radius CS, you deſcribe the Ark *KX*, the ſaid Ark ſhall meaſure the complement of the Latitude in this example 38ᵈ 28′, which being given together with the Sine of the Amplitude CY, it will be eaſie to draw a Scheme of the Analemma, whereby to finde the Suns declination, the time of riſing, and ſetting, his height at ſix the Azimuth thereto, the Vertical Altitude and hour thereto, *&c.* and many other propoſitions depending on the Suns motion, as I have elſewhere ſhewed.

The whole ground hereof is, that a right line extended through the tops of the Sines of any two Altitudes of the Sun taken the ſame day before his declination vary, ſhall meet with the Plain of the Horizon in ſuch a point where the Plain of the Suns parallel interſects the Plain of the Horizon, and finding of two ſuch points, a line drawn through them, muſt needs repreſent the interſection of thoſe two Plains; in the former Scheme the Sines of the two Altitudes are *KM* and IN, a line drawn through the bottomes of thoſe Sines, as *KI* extended ſhall be in the Horizontal Plain, and the Line MN extended through the tops of thoſe Sines is in the plain of the Suns parallel, as alſo in the Horizontal plain; now whether theſe Lines ſtand erect or no is not material, provided they retain their Paralleliſme and due length, and paſs through the points of their Baſes *I*, *K*, for the proportion of the perpendiculars to their Baſes, will be the ſame notwithſtanding they incline to the Horizon.

To Calculate the Latitude, &c. from three ſhadows.

A uſual and one of the moſt troubleſome propoſitions in Spherical Trigonometry, is from three ſhadows to finde the Latitude of the place: thus *Metius* propounds it, and with many operations both in plain and Spherical Triangles reſolves it.

The firſt operations are to finde the Suns 3 Altitudes to thoſe ſhadows, and that will be performed by this proportion,

As the Length of the ſhadow, Is to the perpendicular height of the Gnomon,

So is the Radius, To the Tangent of the Suns Altitude above a Hori-

zontal Plain, which proportion on other Plains will finde the Angle between the Sun and the Plain or Wall.

Next *Metius* gives the diſtances between the points of the three ſhadows, and then by having three Sides of a Plain Triangle, he findes an Angle, to wit, the differences of Azimuth between the reſpective ſhadows, which Angle may be meaſured off the Plain with Chords.

But propounding it thus, Three Altitudes of the Sun above a Horizontal Plain, with the differences of Azimuth between the three ſhadows belonging to thoſe Altitudes, being given, let it be required to finde the Suns true Azimuth, the Latitude of the place, and the Suns Amplitude.

And how this may be Calculated from the former Scheme, I ſhall now ſhew.

In the Triangle ICK of the former Scheme, the two ſides IC and *CK*, repreſent two ſhadows, and the Angle *ICK* is the Angle or difference of Azimuth between them, and the ſaid ſides IC and *CK*, are the Coſines of the Altitudes proper to thoſe ſhadows; now by ſeven Operations in right lined Triangles, we may finde the proper Azimuth or true Coaſt of any of thoſe ſhadows.

1. In the right lined Triangle *ICK*, having the two ſides IC and *CK*, with the Angle between them *ICK*, at one operation may be found both the other Angles *CIK* and *IKC*.

2. In the ſame Triangle by another operation, may be found the Side *IK*.

Then to proceed, draw N f parallel to *IK*, and f M will be the difference of the Sines of both the Altitudes belonging to thoſe ſhadows, whereby may be found KW.

3. The proportion lyes,

As f M the difference of the Sines of both thoſe Altitudes,
Is to f N equal to IK before found,
So is KM *the Sine of the greater Altitude, to* KW *ſought.*

4. 5. 6. By three like Operations may be found in the other ſhadow Triangle, the Angles CKL, and KLC, with the Side KE.

Having proceeded thus far to the Angle IKC, adde the Angle CKL, the ſum is equal to the Angle WKE.

7. Then in the Triangle WKE, we have the two Sides thereof WK, and

and KE given, and the Angle comprehended by them, and at one Operation we may finde both the other Angles EWK, and KEW, the complement of the Angle TWK, is the Angle WKT. The difference between the Angles WKT, and IKC, is the Angle CKZ, which shews the Suns Azimuth from the Meridian proper to that shadow, which may be otherways found, for the difference between EKT and TKC, also shews it.

By two other Operations the Latitude may be found.

1. *As the Radius: Is to WK before found:: So is the Sine of TWK to KT:*
2. *As VK the Sine of the greater Altitude, Is to KT before found, So is the Radius, To the Tangent of the Latitude.*

By another Operation may be found the Amplitude CY, having found the Angle ZKC, the Angle CKp is the complement thereof, then it holds:

As the Radius, Is to CK the Cosine of the greater Altitude,
So is the Sine of the Angle CKp to Cp, the difference between which, and Tp equal to TK, is YC the Sine of the Amplitude sought.

To make any Dyal from three Shadows.

The Geometrical performance of the former Proposition, is insisted upon by *Clavius* in his Book of the *Astrolabe*, but he mentioneth no method of calculation as derivable from it: from this proposition Monsieur *Vaulezard* a French Mathematician educeth a general method for making of Dyals, from three shadows of a Gnomon stuck into a wall at randome, whereof he doth not so much as mention any demonstration; I shall endeavour to deliver the method thereof with as much perspicuity as I can.

Every Plain in some place or other, is an horizontal Plain; admit an Oblique Plain in our Latitude, the Substilar line represents the meridian of that place, and any contingent line drawn at right Angles, thereto will represent a true line of East and West in reference to that horizon, and if the hours did commence at 12, from each side the substile, the Dyal here would shew the true time of the day there.

In every oblique Plain assuming any Point in the Stile, and crossing the Stile with a perpendicular to that Point, which shall meet with the Substile, the Point so found in the Substile, is the Equinoctial Point, in respect of the assumed Point in the Stile; hence we may inferre that if these two Points and the Substilar line were gi-

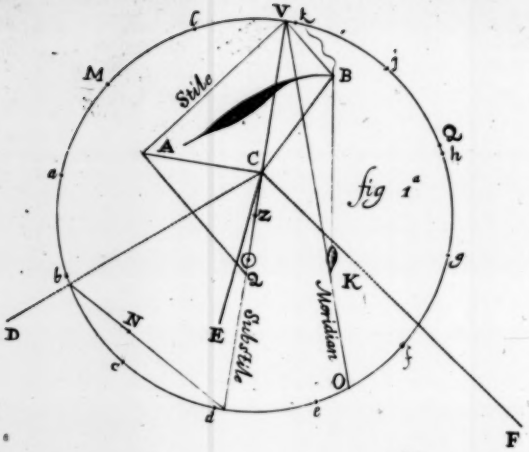
ven the Center of the Dial might be easily found, now the former construction applyed to an oblique Plain, assumed to be an Horizontal Plain, in respect of some unknown place, will find the Equinoctial Point and the substilar Line, the Stile Point being assumed in the extremity of a Gnomon any wayes placed or stuck in a wall at randome.

From the former Scheme it may be observed that the Sine of any of the three Altitudes being erected on the Perpendicular between the foot of that Sine and the Suns parallel, gave an Angle equal to the elevation of the Equinoctial above the Horizon, which is the thing sought in the following work, but the Sine of the greatest Altitude performs the proposition best.

If a stick or pin be stuck into a wall for this purpose, and doth not make right Angles therewith, a Perpendicular must be let fall from the extremity thereof into the wall, which is called the perpendicular Stile, and the distances of the shadowes; from the foot thereof must be measured thence, in respect of the tip of this perpendicular Stile the Equinoctial Point must be found, wherefore it will be convenient to assume the said perpendicular Stile to be the Sine of the greatest of the three Altitudes of the Sun above the Plain, which properly in respect of Us are Angles between the wall and the Sun. Upon this assumption it will follow, that the shortest shadow will be the Cosine of that Angle, and the distance between the tip of the Stile, and the extremity of that shadow will be the Radius: now from the lengths of the three shadows, and the height of the perpendicular Stile, it will be easie to find the Sines and the Cosines of the Angles between the wall and the Sun.

In the following figure, let the Circle be supposed to be described upon a leaning Plain, on which let A B represent a Stile stuck in at randome, the extremity of the three shadows whereto are the points D, E, F, first find the perpendicular Stile thereto C B. and from C draw lines to the three shadowes: now to find the Sines and Cosines of the Angles between the wall and the Sun, assuming C B to be the Sine of the greatest Altitude, make CB in the second figure equal thereto, and perpendicular thereto raise the Line C F, wherein prick down the three shadows CE, CD, & CF, and upon B as a Center with the extent B E describe the Quadrant G A, drawing lines from the three shadowes into B the Center, and the Arks G E, G I, GH, are the three Angles between the wall and the Sun, and the neareſt

distances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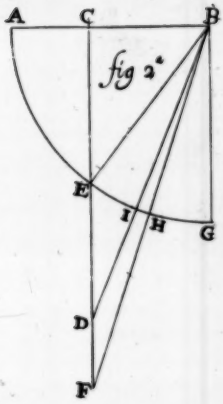
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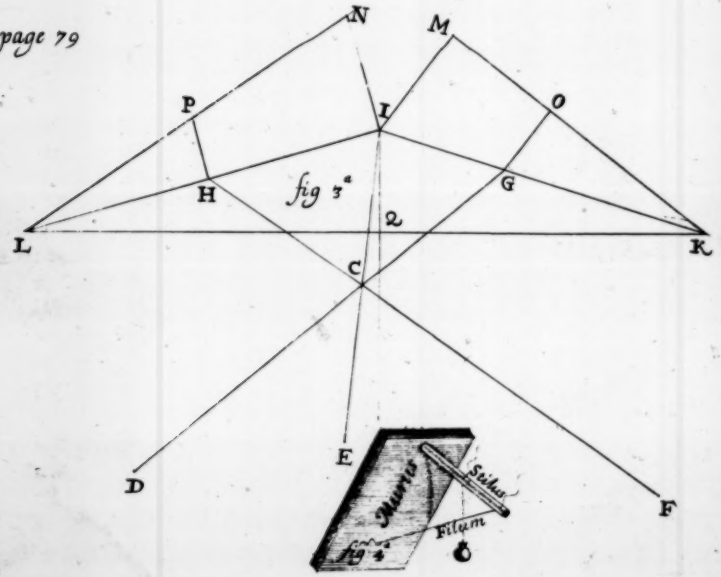
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diſtances from the points E, I, H, being taken to GB, are the Sines thereof, alſo the neareſt diſtances from thoſe Points to AB, are the Coſines thereof.

Being thus prepared, draw three Lines elſewhere as in the third figure meeting in a Center, and making the like Angles as the ſhadows did, and let them be produced beyond that Center, and have the ſame Letters ſet to them: make $\left.\begin{matrix}CH\\CG\end{matrix}\right\}$ in this Scheme, equal to $\left.\begin{matrix}H\\I\end{matrix}\right\}$ the neareſt diſtances from to CB, and make CL equal to EC, in the former Scheme.

Draw the lines IG, IH produced, and make $\left.\begin{matrix}HP\\GO\end{matrix}\right\}$ equal to neareſt diſtances from $\left.\begin{matrix}I\\H\end{matrix}\right\}$ to BG, and parallel to HP and GO, draw the lines IN, IM, each of them made equal to CB, then draw the Hipotenuſals MO, NP, meeting with the Baſes at K and L, draw the Line KL, and it ſhall repreſent the Plains Equinoctial or contingent Line.

From I, let fall the perpendicular IQ produced, and it ſhall be the Subſtilar line on the Plain and the Point Q is the Equinoctial Point ſought.

To place the Subſtile.

Then repair to the Plain whereon you would make the Dyal repreſented by the firſt Scheme, and place GQ therein, ſo that it may make the ſame Angle with the ſhadow CE, as it doth with the Line IC in the third Scheme, and it ſhall be the Subſtilar Line, which is to be produced, alſo prick IQ from the third Scheme from C to Q on the Plain in the Subſtilar Line, and upon the point C, perpendicular to the Subſtile, raiſe the Line AC, and make it equal to CB, drawing the Line AQ, then will the Angle AQC be equal to the complement of the Stiles height; for it is the Angle between the plain and the Equinoctial; juſt as before in the Horizontal plain, the Sine of the greateſt Altitude was erected on a Line falling perpendicular from the foot of the ſaid Sine, to the interſection of the plain of the Suns parallel with the plain of the Horizon, and thereby gave the Angle between the Suns parrallel and the Horizon, equal to the Complement of the Latitude.

To find the Center of the Dyal and Stiles height.

If from the point A you raiſe the Line AV perpendicularly to AQ, where it cuts the Subſtilar Line, as at V, is the Center of the Dial, and he Line AV repreſents the Stile.

3. *To*

3. *To draw the Meridian Line.*

If the Plain recline, a thread and plummet hanging at liberty from the Point *B*, and touching the Plain, will find a Point therein, ſuppoſe K a line drawn from K into the Center of the Dial, ſhall be the Meridian line of the place.

A broad ruler with a ſharp pin at the bottome of it, being in a right line, with a line traced through the length of that ruler, whereto the plummet is to hang, having a hole cut therein for the bullet to play in, will find this Point in a reclining plain.

On an inclining Plain, if the thread and plummet hang upon the Stile at liberty, to ſome part of the Stile more remote from the Center, faſten another thread, which being extended thence to the Plain juſt touching the former thread and plummet hanging at liberty, will find many Points upon the Plain, from any of which, if a line be drawn into the Center it ſhall be the Meridian line required; See the fourth figure: or in either of theſe caſes hold a thread and plummet ſo at liberty that it may juſt touch the Stile, and bringing your ſight ſo, as to caſt the ſaid thread upon the Center at the ſame time by the interpoſition of the thread; the eye will project a true Meridian-line on the Plain, for the thread repreſents the Axis of the Horizon, and the Plain of the Meridian, is in the ſaid Axis.

If the Center fall inconvenient upon the Plain, it will be neceſſary to draw the Plains perpendicular, paſſing through the foot of the perpendicular Stile C, and meaſure the Angle of ſome of the Shaddowes from it, and accordingly ſo place it in the third figure or draught on the floor, on which find the Center, and afterwards aſſign the Center on the Plain where it may happen convenient, and from the ſaid draught, by help of the Plains perpendicular, ſet off the ſubſtile and Stiles height; On plains on which probably the Stile hath but ſmall height, the perpendicular Stile in this work muſt be aſſumed the ſhorter.

4. *To draw the Hour Lines.*

Theſe may be ſet off in a Paralellogram after the Subſtile, Stile and Meridian are placed which I have handled before.

Or they may be inſcribed by the Circular work, if we aſſume the perpendicular Stile A C, to be the Sine of the Stiles height, the Radius

dius thereto will be A V, prick the ſaid Radius on the Subſtile from V the Center to, Z, and upon Z as a Center, deſcribe the Circle as in the firſt figure, and find the regulating point ⊙ by former directions, over which from O the point where the Meridian of the place cuts the ſaid Circle, lay a ruler, and it finds M, on the oppoſite ſide; being the Point from whence the Circle is to be divided into 12. parts, for the houres with ſubdiviſions for the halfes & quarters. Here note that the arch M V, is the inclination of Meridians, the whole Semicircle being divided but into 90^{d}, and if that be firſt given (as hereafter) we may thereby find the Point O, whence the Meridian Line is to be drawn into the Center.

5. *A Method of Calculation ſuited hereto.*

This is altogether the ſame as for finding the Azimuth of the ſhadowes on the Horizontal Plain and as eaſie, whereby in the third figure, the Angle of the Subſtile C I Q, muſt be ſet off from the ſhadow I C E, juſt as the Meridian Line C P, might be ſet off from the ſhaddow C K on the Horizontal.

And the complement of the Stiles height A Q V, in the firſt figure here, as the complement of the Latitude K T V, was found there.

Laſtly, for placing the Meridian Line of the place by calculation, the Subſtiles diſtance from the plains perpendicular, and the reclination of the Plain muſt be found, which may be got eaſily at any time without dependence on the Sun, and then in the often-mentioned oblique Triangle in the Sphere, we have two ſides with the Angle comprehended given, to wit, the complement of the Stiles height, the complement of the reclination, and the Subſtiles diſtance from the Plaines perpendicular whereby may be found the inclination of Meridians: and conſequently the Meridian line of the place, alſo the Latitude thereof, with the Plains declination, if they be required.

To perform all this other wiſe Geometrically and inſtrumentally, (but not by calculation) there was an entire conſiderable quarto treatiſe, with many excellent Prints from braſs-Plates thereto belonging, Printed at Paris in *France*, in *An.* 1643. by Monſieur *Deſargues* of *Lions*, which Treatiſe I have ſeen but not peruſed, a year after was publiſhed the ſmall Treatiſe of Monſieur *Vaulezard* before mentioned.

The

The inſcribing of the Signes, Azimuths, parallels of the longeſt day, &c. are lately handled by Mr. *Leybourn* in his Appendix to Mr. *Stirrups* Dyalling, as alſo by Mr. *Gibſon* in his Algebra, who thinks in many caſes that they deform a plain, and are ſeldome underſtood by the vulgar, wherefore it will not be neceſſary to treat thereof.

The directions throughout this Book are ſuited to the Northern Hemiſphere and are the ſame in the Southern Hemiſphere, if the words South for North, and North for South be mutually changed.

Since the Printing of this Treatiſe, I have not had time to reviſe it with the Copy, and ſo cannot give thee a full account of what faults may have eſcaped, which I think are not many; theſe few following be pleaſed to correct.

Errata.

Page 17 line 3 *upon* M read *upon* C. *p.* 15 l. 34 *for Subſtilar Sine* r. *Subſtilar Line. p.* 25 l. 12 *for* 20^{d} r. 15^{d}. *p.* 36 l. 17 *for* 51′. r. 31′.

FINIS.